

# Quantum reference frames for general symmetry groups

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**Abstract:** Treating reference frames as physical systems, subject to the laws of quantum mechanics, they become quantum reference frames. Located at the interplay of quantum and gravitational physics, their treatment marks an essential step towards the construction of a relational quantum theory. In this work, we introduce a relational formalism which identifies coordinate systems with elements of a symmetry group  $G$ . We define a general operator for reversibly changing between quantum reference frames associated to a group  $G$ . This generalises the known operator for translations and boosts [1] to arbitrary finite and locally compact groups, including non-Abelian groups.

**Preprint:** <https://arxiv.org/abs/2004.14292>

**Keywords:** quantum foundations, quantum reference frames, relational quantum mechanics, quantum gravity

Reference frames are essential in the description of physical phenomena. Whenever a physical quantity is measured or a physical event is described, this is done with respect to a specified reference frame. Treating reference frames themselves as physical objects and submitting them to the laws of quantum mechanics, they become quantum reference frames [2-5]. Recently, there has been an increased interest in analysing spatial and temporal quantum reference frames and in establishing a formalism that allows to switch between different perspectives [1, 6-10]. Some of the most striking results can be found in Giacomini et al. [1] in which a formalism is developed to deal with quantum reference frame changes in the case of the Galilean group. The present work is partially based on these ideas while aiming to generalize beyond this specific group to general symmetry groups.

## 1 Entanglement and superposition: reference-frame dependent properties

By identifying coordinate systems with elements of a symmetry group  $G$ , we define a general operator for reversibly transforming between different quantum reference frames associated to a group  $G$ . We show under which conditions one can uniquely assign coordinate choices to physical systems to form reference frames and how to reversibly transform between them, providing transformations between coordinate systems which are ‘in a superposition’ of other coordinate systems. The relational formalism constructed in this work is based on two principles: the principles of relational physics and of coherent change of reference frame. As a consequence, we find that quantum properties such as entanglement and superposition are not absolute but depend on the reference frame relative to which they are described. As such, the conclusions of Giacomini et al. [1] are shown to be generic to quantum reference frames for arbitrary groups.

## 2 Imperfect reference systems and irreversible changes

We prove a theorem stating that the change of quantum reference frames consistent with the two main principles of this work is unitary if and only if the reference systems carry the left and right regular representations of  $G$ . We also define irreversible changes of reference frame for classical and quantum systems in the case where the symmetry group  $G$  is a semi-direct product  $G = N \rtimes P$  or a direct product  $G = N \times P$ , providing multiple examples of both reversible and irreversible changes of quantum reference systems along the way.

## 3 Application to the Wigner’s friend thought experiment

Finally, we apply the relational formalism and changes of reference frames developed in this work to the Wigner’s friend scenario [11], finding similar conclusions to those in relational quantum mechanics [12], using an explicit change of reference frame as opposed to indirect reasoning using measurement operators.

Future work will involve extending this relational formalism to the perspective-neutral framework [9, 10] and applying it to other thought experiments involving encapsulated observers [13].

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