Generalized quantum steering ellipsoids for qubit-field system

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Abstract

Quantum steering in a system consisting of a qubit coupled to a single-mode field is studied[1] when classical-like measurements are implemented through heterodyne detection schemes on the field mode. Measurements on the field mode that collapse the state of the field onto coherent states are considered with the aim of constructing the generalized quantum steering ellipsoid for the qubit. The quantum steering ellipsoid was introduced in[2] as a means of visualizing the state space of two qubits using only the three-dimensional Bloch-ball picture of single-qubit states. The ellipsoid corresponds to the set of all states into which one of the qubits in a two-qubit state can be steered to, through all possible measurements on the other. In this work, we have extended the quantum steering ellipsoid construction to the case where the second qubit is replaced by a single field mode by implementing heterodyne detection on the field mode. We have used diagonal state representation[3, 4] for the field and obtained the closed-form expressions for the components of the Bloch vector of the qubit corresponding to a given outcome for the measurement on the field mode. We analyzed various states of the joint qubit field system, and it is found that the steering ellipsoid is the full Bloch sphere when the joint state is pure except when it is a product state. Our results showed that in some cases, the steering set does not form an ellipsoid since only heterodyne detection is considered. This is particularly the case when the initial qubit-field state is mixed. we explored the structure of the steering set for the mixed state and a representative example is shown figure.1 We studied the evolution of the steering ellipsoid corresponding to joint evolution of the qubit and field under Jaynes-Cummings Hamiltonian. We found that steering ellipsoid for qubit field system is independent of the amount of entanglement between the two systems. The entanglement between the qubit and field is found to be smoothly varying function while the steerability jumps discontinuously between full and no steerability corresponding to pictorially to the full sphere and a single point respectively.

In this work, we propose that the steerability established in the qubit-field system can be used in performing one sided device-independent quantum key distribution (1S-DI-QKD). It is known[5] that the ability to do steering as indicated by the violation of corresponding steering inequalities also indicates that it is possible to get a finite key rate from 1S-DI-QKD. In the one sided scheme the apparatus operated by one of the parties, say Alice, in the QKD is suspected to be compromised and possibly completely under the control of the eavesdropper. The measurement and other apparatus on the other (Bob's) side is trusted. With these restrictions, in previous versions of 1S-DI-QKD the key distribution is done using pairs of photons distributed by common source. The asymmetry in the conception of 1S-DI-QKD naturally lends itself to a scenario like the one we have considered. Instead of using a pair of photons, Bob can have a qubit at hand which can get entangled with a photon (or a weak pulse of light) through the JaynesCummins or any other suitable interaction. Bob can prepare multiple copies of such qubit-field systems and the photonic part can be sent to Alice for implementing the 1S-DI-QKDprotocol as in [5]. Steerability can be ascertained using a scheme involving heterodyne detection as described in this work. A non-trivial generalized steering ellipsoid for the qubit-field states that are prepared by Bob will be indicative of the ability to implement the protocol with non-zero key rate.

Keywords

quantum steering ellipsoid, heterodyne detection, qubit-field system, 1S-DI-QKD

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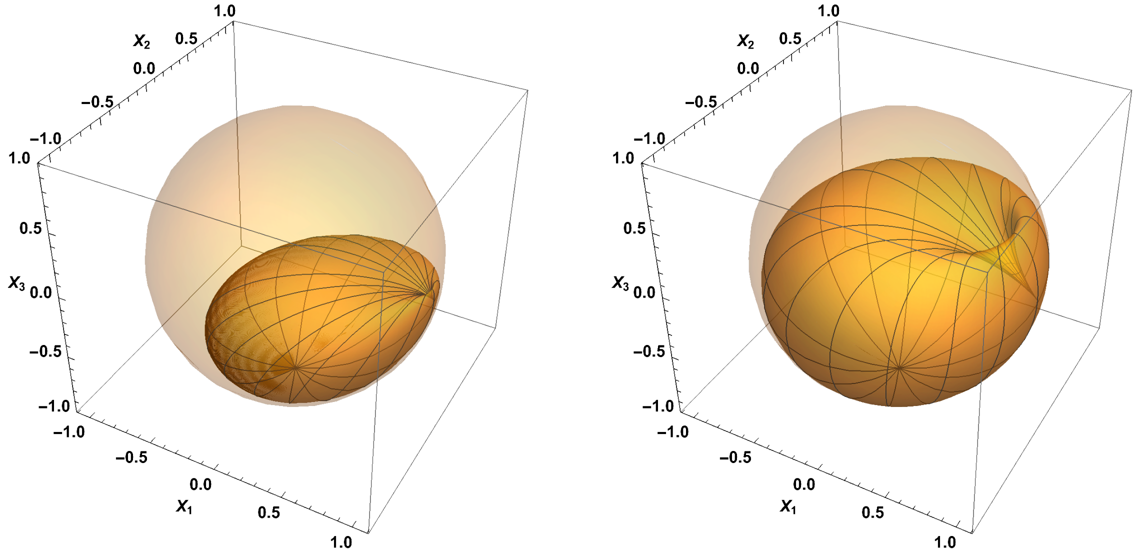

Figure 1: The steering set for the mixed state. The left figure corresponds to $p = 0.3$ and the right figure corresponds to $p = 0.9$. The full Bloch sphere is also shown for reference.

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