Resource-efficient fault-tolerant quantum computation with optical hybrid states

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Abstract. We propose an all-linear-optical scheme to ballistically generate a cluster state for measurement-based topological fault-tolerant quantum computation (QC) using hybrid photonic qubits entangled in a continuous-discrete domain. In the presence of photon losses, we show that our scheme leads to a significant enhancement in both tolerable photon-loss rate and resource overheads. We report a photon-loss threshold of $\sim 3.3 \times 10^{-3}$, which is higher than most of the known schemes. Furthermore, resource overheads to achieve logical error rate of $10^{-6}(10^{-15})$ is estimated to be $\sim 8.5 \times 10^5 (1.7 \times 10^7)$ which is significantly less by multiple orders of magnitude compared to other reported values.

Keywords: Measurement based quantum computation, topological quantum error correction, hybrid-qubits, photon-loss threshold.

Introduction: Here, we propose an all-linear-optical measurement-based fault-tolerant hybrid topological QC (HTQC) scheme on 3D lattice $|\mathcal{C_L}\rangle$ [1] of hybrid qubits. $\{|\alpha\rangle |\mathrm{H}\rangle, |-\alpha\rangle |\mathrm{V}\rangle\}$ for a basis for hybrid-qubits, where $|\pm\alpha\rangle$ are continuous variable (CV) coherent states of amplitude α, and $|\mathrm{H}\rangle$ and $|\mathrm{V}\rangle$ are discrete variable (DV) single-photon states with horizontal and vertical polarizations in Z direction. Crucial to our scheme is a near-deterministic hybrid Bell-state measurement (HBSM) on the hybrid qubits using photon number parity detectors (PNPD) and on-off photodetectors (PD). The HBSMs are used as entangling operations in building a $|\mathcal{C_L}\rangle$. In HTQC, we only need HBSMs acting on 3-hybrid-qubit cluster states, which are considered to be offline resource states without the need of active switching and feed-forward operations. In this sense, our scheme is *ballistic* in nature provided the outcomes of HBSMs are noted to interpret the measurement results during the quantum error correction and QC. Both CV and DV modes of a hybrid qubit support the HBSMs to build the $|\mathcal{C_L}\rangle$, while only the DV mode suffices for the QEC [2].

Noise model: Let η be the photon-loss rate due to imperfect sources and detectors, absorptive optical components and storage. The effect of photon loss is three-fold [3]: (i) leads to dephasing of hybrid qubits with rate $p_Z = [1-(1-\eta)\,\mathrm{e}^{-2\eta\alpha^2}]/2$, (ii) lowers the success rate of HBSM $p_f = (1+\eta)\mathrm{e}^{-2\alpha'^2}/2$, where $\alpha' = \sqrt{1-\eta}\alpha$ and (iii) hybrid qubits leak out of logical basis. Also, during photon loss HBSM introduces additional dephasing.

Results: The value of photon loss threshold η_{th} for $0.8 \le \alpha \le 2$ is $\mathcal{O}(10^{-3})$, which is an order greater than the hybrid-qubit-based QC (HQQC) [3], coherent state QC (CSQC) [4] and topological photonic QC (TPQC) [5]. Multi-photon qubit QC (MQQC) [6], parity state linear optical QC (PLOQC) [7] and error-detecting quantum state transfer based QC (EDQC) [8] provide η_{th} which is less than HTQC, Fig. 2(a). For computational error rate as low as 8×10^{-5}, $\eta_{\mathrm{th}} = 0$ for optical cluster-state QC (OCQC) [9]. Thus, HTQC outperforms the OCQC.

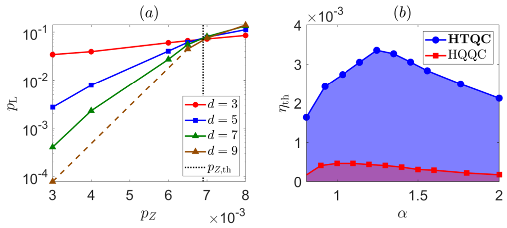

Figure 1: (a) Logical error rate p_{L} is plotted against p_Z for $\alpha = 1.25$. The intersecting point of these curves correspond to threshold dephasing rate $p_{Z,\mathrm{th}}$. (b) Tolerable photon loss rate η_{th} is plotted against α. Compared to non-topoligical HQQC [3], HTQC has an order of higher value for η_{th}.

Resource overheads: On average $N = 1125d^3/[4(1-\mathrm{e}^{-2\alpha'^2})^2]$ hybrid qubits are incurred to build $|\mathcal{C_L}\rangle$ of side d. For optimal $\alpha \approx 1.25$, $N \approx 8.5 \times 10^5 (1.7 \times 10^7)$ to achieve $p_{\mathrm{L}} \sim 10^{-6}(10^{-15})$.

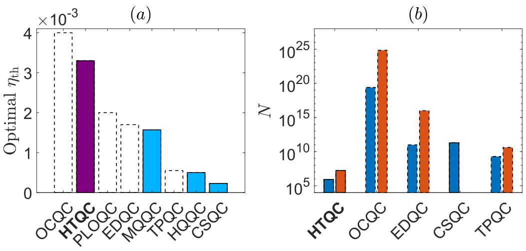

Figure 2: (a) η_{th} for various optical QC schemes. η_{th}'s of OCQC, PLOQC and EDQC (dashed borders) are valid only for zero computational error. (b) Resource overhead N to achieve $p_{\mathrm{L}} \sim 10^{-6}$(blue shorter bars) and $p_{\mathrm{L}} \sim 10^{-15}$(orange taller bars). HTQC is practically favorable for large scale QC both in terms of η_{th} and N. There are no published reports about N to attain $p_{\mathrm{L}} \sim 10^{-6}(10^{-15})$ for PLOQC, MQQC, TPQC and HQQC.

Conclusion: HTQC is practically favorable for large scale QC both in terms of photon loss rate threshold η_{th} and incurred resources N.

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