Experimental few-copy multipartite entanglement detection

V. Saggio, A. Dimić, C. Greganti, L. A. Rozema, P. Walther, and B. Dakić

As technological advances allow for the realization of increasingly big quantum states, a method for detecting entanglement in a reliable and resource-saving way is urgently needed. The most used approach is based on measuring expectation values of witness operators [1], that confirm the presence of entanglement when measured to be negative. However, measurements of expectation values require a sufficient amount of detection events, or equivalently copies of the quantum state. Thus, this approach results inconvenient when only a small number of detection events is available. This difficulty can be overcome by treating entanglement detection as a probabilistic procedure [2].

Given a certain state in which we want to verify entanglement, we can write its corresponding witness operator in form of observables O_j, with $j=1,\ldots,L$. We can then randomly draw one observable and apply it to the state, and repeat this procedure N times. These observables are written in binary form, and therefore return an outcome 1 or 0 only. 1 (0) is associated with the success (failure) of the measurement. Since we use only a single copy per measurement to extract the corresponding binary outcome, N is therefore the number of copies used. The probability to obtain a successful outcome is maximized to a certain entanglement value p_e for our state, and upper bounded by a separable bound p_s for any separable state, with $p_e > p_s$. Defining $\delta=p_e - p_s$, and assuming that when applying the O_js to the state N times we obtain d successful outcomes, the experimental entanglement value can be calculated as d/N. This leads to $\delta=d/N - p_s$. It has been shown in [2] that the probability $P(\delta)$ to obtain $\delta>0$ for an arbitrary separable state is upper bounded as $P(\delta)<e^{-D(p_s+\delta||p_s)N}$, which goes exponentially fast to zero with N ($D(x||y)$ is the Kullback-Leibler divergence). This means that the confidence for entanglement detection reads $C(\delta) =1-P(\delta)$, and is lower bounded by $Cmin(\delta) =1-e^{-D(p_s+\delta||p_s)N}$, which grows exponentially fast in N. This is the key of our method.

The goal is to experimentally validate this theory by detecting entanglement in an experimental six-qubit cluster state $|Cl_6\rangle$, generated using a multi-photon source composed of three identical single-photon sources [3]. To detect entanglement in this state, we need a proper set of O_js and the value of the separable bound p_s. For this reason, we rewrite the witness operator $W_{Cl6}=I-|Cl_6\rangle\langle Cl_6|$ [4] in terms of binary observables $(I+S_j)/2$, where the operators S_j are the generators of the state, with $j=1,\ldots,64$. The separable bound p_s reads 3/4 and is derived analitically. After randomly sampling the 64 binary observables $N=160$ times and applying them to our cluster state, the obtained outcomes are used to calculate the deviation δ. If δ is positive, genuine six-qubit entanglement is detected with at least confidence $Cmin(\delta)$. Interestingly, entanglement is detected with at least 97% confidence with only 50 copies of our cluster state.

This approach entails a significant reduction of resources, and therefore might prove essential in multi-photon experiments with only a limited number of copies available.

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