History state formalism for relativistic particles

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A properly defined time operator, based on an enlarged Hilbert space $\mathcal{H}$, is employed to embed the Dirac and Klein Gordon equations within a formalism where covariance is an explicit quantum symmetry [1, 2]. In both cases, it follows a natural definition of Lorentz operators which induce the usual transformation properties of the wave functions and explicitly preserve the four-dimensional inner product of the corresponding $\mathcal{H}$. In this setting, a particular physical theory is introduced through a timeless Wheeler DeWitt-like equation, whose projection onto space-time coordinates leads to the Dirac (Klein-Gordon) equation. These invariant constraints are eigenvalues equations for mass operators which are hermitian in the four-dimensional products. Remarkably, the ensuing orthogonality of different eigenstates implies the usual three-dimensional inner product of Dirac (Klein Gordon) for states of definite mass. This relation allows a straightforward computation of expectation values of observables at a given time in a given frame of reference, completing the connection with the usual formulations. In the scalar case, it also provides a positive definite norm, and hence a Hilbert space for the Klein Gordon equation, a lacking aspect of the conventional formulation. In addition, the formalism allows the superposition of different theories (e. g. mass superposition) and remains valid in the presence of fixed external fields, revealing special orthogonality relations. Other details such as related quantization techniques in quantum gravity [3] and the non-relativistic limit, with its connection to the Page and Wootters formalism [4, 5], are discussed. The corresponding second quantization formulation is also introduced.

References

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