Analysing causal structures using Tsallis entropies

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Abstract. We analyse causal structures using Shannon and Tsallis entropies with and without post-selection. Without post-selection, Shannon entropies are known to have certain limitations for certifying non-classicality. We investigate whether Tsallis entropies can overcome these limitations and derive constraints on Tsallis entropies implied by a general causal structure before applying these to find new entropic inequalities for the Triangle causal structure. In the post-selected Bell causal structure, we find numerical evidence that Shannon and Tsallis entropic inequalities are insufficient for detecting non-classicality in the 2 input and 3 outcome case, in contrast to a known result for the 2 outcome case.

Keywords: Causal structures, Tsallis entropies, Non-classicality, Entropic Inequalities

BACKGROUND: Classical and quantum physics may impose different constraints on correlations arising from a causal structure. Exploiting this difference to certify non-classicality of observed correlations in a causal structure has provided useful foundational insights as well as technological applications. The sets of classical and non-classical correlations in simple causal structures with small cardinality of variables are relatively well understood but characterising these sets in general is a difficult problem [1]. One well-studied approach employed for tackling this problem is the entropy vector method that has proven to be useful in some cases [2]. However, the entropy vector method for analysing causal structures using Shannon entropies has been found to have limitations in certifying non-classicality for certain causal structures [3]. A natural question is whether the use of generalised entropies can overcome these limitations. A further technique can also be employed to obtain tighter entropic inequalities in causal structures with parentless nodes such as the bipartite Bell scenario causal structure, that involves generating a new causal structure by post-selecting on values of such nodes [4]. In the post-selected Bell causal structure with two inputs and two outputs per party, there exists a method for identifying all non-classical (i.e., CHSH-violating) correlations using Shannon entropic inequalities [5]. This method involves taking the observed distribution, post-processing it using a non-classicality non-generating (NCNG) operation, and then checking for the violation of an entropic inequality. However, previous work did not investigate the extension to more outputs. Here, we analyse causal structures with and without of post-selection and using Shannon as well as Tsallis entropies. Exploring the above mentioned questions, we produce the following results.

RESULTS 1: Without post-selection [6]

1. We derive the constraints on classical Tsallis entropies of random variables that are implied by the causal structure between them. These encode conditional independences of the causal structure and upper bound the classical Tsallis conditional mutual information between the variables involved.
2. We find that in the classical case, 53 and 126 Tsallis causal constraints are required to characterise all conditional independence relations in the Bell and Triangle causal structures, which have 5 and 6 nodes respectively. In contrast, only a maximum of n Shannon entropic constraints are required for an n-node causal graph. Further the former are inequalities while the latter are equality constraints.
3. Due to the nature of the new constraints mentioned in 2., we find that the computational procedure of the entropy vector method becomes too time consuming even for relatively small causal structures. Instead, we find new Tsallis entropic inequalities for all q>=1 by using known Shannon entropic inequalities for the Triangle causal structure [2] and obtaining new, q-dependent lower bounds subject to our causal constraints of 1. and the monotonicity and strong-subadditivity constraints obeyed by (classical) Tsallis entropies for q>=1 [7].

RESULTS 2: With post-selection [8]

1. We consider the 2 input 3 outcome Bell scenario and show that the analog of the method considered in [5] for the 2 outcome case cannot certify non-classicality in all cases. We identify a class of non-classical distributions in this scenario whose non-classicality cannot be detected through this method with respect to both Shannon and Tsallis entropic inequalities.
2. We consider natural extensions of this method and give numerical evidence that these do not help either. Our evidence suggests that even if we allow the observed correlations to be post-processed according to a more general class of NCNG operations than those of [5], entropic inequalities for either the Shannon or Tsallis entropies cannot detect the non-classicality. This indicates that entropic inequalities are generally not sufficient for detecting non-classicality in the Bell causal structure.
3. The main region of interest here is the set of all non-signalling distributions in the 2 input 3 outcome Bell scenario that satisfy all the CHSH-type inequalities, since the method of [5] always applies for CHSH-violating distributions. This region is a polytope, we find its vertex description and show that any non-classical distribution in the polytope violates exactly one I2233 [9,10] type inequality. We provide examples of non-classical distributions in this polytope that can also be identified using entropic inequalities under suitable NCNG post-processings. However, our results 1. and 2. suggest that not all such non-classical distributions can be identified using Shannon/Tsallis entropic inequalities even under a general class of NCNG operations.

CONCLUSION: Our results reveal new mathematical properties of classical and quantum Tsallis entropies and of non-classical distributions in the 2 input 3 outcome Bell scenario. They also indicate the drawbacks of entropic approach for analysing causal structures, both with and without post-selection.

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