Macro-to-micro quantum mapping and the emergence of nonlinearity

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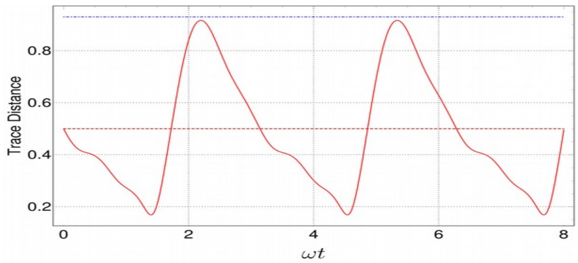
Introduction. As a universal theory of physics, quantum mechanics must assign states to every level of description of a system, and also describe the interconnections among them. Assuming that we only have a coarse-grained access to a macroscopic system, here we show how to assign to it a microscopic description that abides by all macroscopic constraints. The present work [1] taps on the theory of coarse-graining (CG) maps [2-12] in order to consistently describe how quantum mechanics deals with the *macro-to-micro* and *micro-to-macro* assignments, even when the distinction between system and environment is not clear cut like in the usual open quantum system paradigm [13-15]. As a by-product, we show how effective nonlinear dynamics can emerge from the linear quantum evolution, and we readily apply it to a state discrimination task. Hitherto, effective quantum nonlinear dynamics [16-18], lacked a general framework. Given the ubiquity of nonlinear processes within the macro world, such a general microscopic understanding is as desirable as necessary.

Averaging assignment. In general lines, given an arbitrary set of macroscopic observations, our method gives an ensemble of microscopic states which the underlying physical system could be in. Such an approach is grounded on two basics premises. The first one is that all of our concrete perception of nature invariably arises through measurement processes. The second assumption is that measurements of macroscopic systems are inherently coarse-grained, with "classical" features emerging due to an effective CG description of quantum systems. Based on these premises we construct an *averaging assignment* method. Considering a macroscopic level of description being a CG version of a quantum microscopic level modeled by a CG map $\Lambda : \mathcal{L}(\mathcal{H}_D) \to \mathcal{L}(\mathcal{H}_d)$ (CPTP map, with $\dim(\mathcal{H}_D) > \dim(\mathcal{H}_d)$) and an arbitrary system characterized in this macroscopic level by a set of observations $\mathcal{O} = \{o_i\}$, so we define an *averaging map* $\mathcal{A}_\Lambda : \mathcal{O} \to \mathcal{L}(\mathcal{H}_D)$ which assigns to this macroscopic observation $\mathcal{O}$ a microscopic state, which is the ensemble-average of the set all microscopic states that are compatible with that observations. This assignment map $\mathcal{A}_\Lambda$, in general is not CP nor linear, but these characteristics pose no problems as discussed in the paper [1].

Effective CG dynamics. As a by-product of our two-way generalization of the open quantum system paradigm, we establish the effective dynamics that can arise from the microscopic unitary quantum dynamics. Such an effective dynamics model assumes only access to the macroscopic description of a system, to a model of the microscopic dynamics and to the CG map describing our ability to construe the microscopic world. More explicitly, given an initial effective CG state $\rho(0) \in \mathcal{L}(\mathcal{H}_d)$, a microscopic unitary evolution map $\mathcal{U}_t : \mathcal{L}(\mathcal{H}_D) \to \mathcal{L}(\mathcal{H}_D)$, and the CG map Λ, the family of effective channels $\Gamma_t : \mathcal{L}(\mathcal{H}_d) \to \mathcal{L}(\mathcal{H}_d)$ such that for each time t the evolved effective state is given by $\rho(t) = \Gamma_t[\rho(0)]$ is obtained as $\Gamma_t = (\Lambda \circ \mathcal{U}_t \circ \mathcal{A}_\Lambda)$. Since $\mathcal{A}_\Lambda$ is nonlinear in general, from this effective dynamical approach it becomes clear how nonlinear dynamics can effectively emerge from the unitary linear quantum evolution.



Figur 1: Effective distance evolution. *The red solid line describes the distance evolution* $\mathcal{D}(\rho(t),\chi(t)) = |\rho(t) - \chi(t)|/2$ *between two CG states* $\rho(t)$ *and* $\chi(t)$*; the red dashed line represents the initial distance* $\mathcal{D}(\rho(0),\chi(0))$*; while the blue dot-dashed line shows the distance between the underlying microscopic assignments* $\mathcal{D}(\mathcal{A}_\Lambda[\rho(0)] - A_\Lambda[\chi(0)])$. *For details about the states and the unitary, see the article* [1].

Applications. Among the applications, we model emergent nonlinear dynamics in the blurred detection of cold-atoms in optical lattice scenario[1-3,19-22], in which increasing distance of two initial CG states can be achieved, even under rather simple local microscopic dynamics – see Fig 1. We expect that such a result may be useful in quantum computation applications, e.g., in quantum error corrections protocols.

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