On the properties of quantum abstract detecting systems

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In a recent paper [1], quantum abstract detecting systems (QADS) were introduced as a framework for the definition and study of quantum algorithms for the detection task (i.e., deciding whether a boolean oracle returns true for at least one input). The main definition is the following:

Definition 1 *A* quantum abstract detecting system (QADS) *is any (classical deterministic) algorithm that takes as input a boolean function (given by a circuit)* $f : \{0,1\}^k \to \{0,1\}$ *from a set of inputs* $\mathcal{M}$ *and outputs a unitary transformation* $U = U(f)$ *on a Hilbert space* $\mathcal{H}$ *whose dimension only depends on* k, *together with a state* $|\psi(0)\rangle \in \mathcal{H}$ *(that only depends on* k *too) such that*

$$\{x \in \{0,1\}^k \mid f(x) = 1\} = \emptyset \implies U|\psi(0)\rangle = |\psi(0)\rangle$$

The transformation U *will be called* detecting operator *and* $|\psi(0)\rangle$ *is known as the* initial state.

In [1], it is proved that many quantum algorithms for search and detection (including Grover's algorithm, several families of quantum walks and the quantum abstract search algorithm, among others) fall under this new formalism. Also, general ways of studying their performance are defined.

One advantage of this approach is that it allows to easily define new QADS from existing ones. For instance, if U_1 and U_2 are detecting operators of two QADS with the same initial state $|\psi(0)\rangle$, then we can define a new QADS with detecting operator given by $U(f) = U_1(f)U_2(f)$ and initial state $|\psi(0)\rangle$. This leads to the construction of new algorithms that show improved detection rate in some cases.

References

[1] E.F. Combarro, J. Ranilla, and I.F. Rúa. Quantum abstract detecting systems. *Quantum Information Processing*, 19 (8), 2020. `https://link.springer.com/article/10.1007/s11128-020-02763-w`.