Quantum Advantage in Information Retrieval

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Random access coding involves the encoding of a random input string into a shorter message string. The encoding should be such that any element of the input string can be retrieved with high probability from the message string. Such tasks have long been studied as examples in which the communication of quantum information can provide advantage, i.e. enhanced performance, over classical information, e.g. [1, 3, 6, 8, 10, 15, 18, 19].

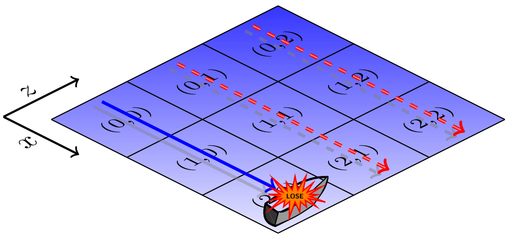

Figure 1. The Torpedo Game is a pacificist alternative to *Battleship* where the aim is to avoid sinking Alice's ship, depicted here in dimension 3.

However, random access coding concerns only one kind of information retrieval. In this work we introduce another such task – the Torpedo Game (see Fig. 1). It is similar to random access coding, but with additional requirements involving the retrieval of relative information about elements of the input string. Taking a geometric perspective it may also be viewed as a pacifist version of the popular strategy game *Battleship*. Quantum strategies outperform classical strategies for the Torpedo Games with bit and trit inputs. In particular, quantum perfect strategies exist in the trit case and provide a **greater quantum advantage than for the comparable random access coding task** [19].

Optimal quantum strategies emerge from an analysis in terms of the discrete Wigner function. Wigner negativity is a signature of non-classicality in quantum systems that is related to contextuality and that has been widely studied as a resource for quantum speed-up and advantage [4, 5, 7, 9, 12, 14, 16, 20]. Knowing which characteristic lies at the source of better-than-classical performances can both allow for comparison of quantum systems in terms of their utility, and offer a heuristic for generating further examples of quantum-enhanced performance. Our optimal quantum strategies are indeed Wigner negative, with **perfect quantum strategies derived from maximum Wigner negativity**. We exploit the fact that for $x, z \in \mathbb{Z}_d$:

$$\mathrm{Tr}\left[|\psi_{x,z}\rangle\langle\psi_{x,z}|\,(A_{x,z} + \mathbb{I})\right] = 0 \tag{1}$$

where $A_{x,z}$ is the phase-point operator at $(x, z) \in \mathbb{Z}_d^2$ and $|\psi_{x,z}\rangle$ is an eigenvector of $A_{x,z}$ with eigenvalue -1. These states display Wigner negativity. Yet while negativity is necessary for advantage in the Torpedo game, it is not sufficient.

To more precisely pinpoint the source of quantum advantage we must look further. One candidate is preparation contextuality [17], another signature of non-classicality that has been linked to QRACs in numerous studies [2, 6, 18]. It has been shown to be necessary for advantage in a restricted class of random access codes subject to an obliviousness constraint [11, 16].

In this work, however, we focus on a different characteristic called sequential contextuality [13]. It indicates the absence of a hidden variable model respecting the sequential structure of a given protocol. Subject to an **assumption of bounded-memory**, we find that this characteristic is **necessary and sufficient for quantum advantage**, not just in random access coding but **in any information retrieval task** expressible in a sequential form. Moreover, we show that it **quantifies the degree of advantage** that can be achieved:

Theorem. *Given any information retrieval task expressible in a sequential communication scenario, and strategy with empirical behaviour e,*

$$\varepsilon \geq \mathsf{NCF}(e)\nu$$

where ε is the probability of failure, averaged over inputs and questions, $\mathsf{NCF}(e)$ is the noncontextual fraction of e with respect to a dit ontology with d fixed by the communication scenario, and $\nu := 1-\theta^C$ measures of the hardness of the task (where θ^C is the classical value of the task).

[1] Edgar A Aguilar, Jakub J Borkała, Piotr Mironowicz & Marcin Pawłowski (2018): **Connections between mutually unbiased bases and quantum random access codes**. *Physical review letters* 121(5), p. 050501, doi: 10.1103/PhysRevLett.121.050501.

[2] Andris Ambainis, Manik Banik, Anubhav Chaturvedi, Dmitry Kravchenko & Ashutosh Rai (2019): **Parity oblivious d-level random access codes and class of noncontextuality inequalities**. *Quantum Information Processing* 18(4), p. 111, doi:10.1007/s11128-019-2228-3.

[3] Andris Ambainis, Ashwin Nayak, Ammon Ta-Shma & Umesh Vazirani (1999): **Dense quantum coding and a lower bound for 1-way quantum automata**. In: *Proceedings of the thirty-first annual ACM symposium on Theory of computing*, pp. 376–383.

[4] Juan Bermejo-Vega, Nicolas Delfosse, Dan E Browne, Cihan Okay & Robert Raussendorf (2017): **Contextuality as a resource for models of quantum computation with qubits**. *Physical review letters* 119(12), p. 120505, doi: 10.1103/PhysRevLett.119.120505.

[5] Lorenzo Catani & Dan E Browne (2018): **State-injection schemes of quantum computation in Spekkens' toy theory**. *Physical Review A* 98(5), p. 052108, doi:10.1103/PhysRevA.98.052108.

[6] André Chailloux, Iordanis Kerenidis, Srijita Kundu & Jamie Sikora (2016): **Optimal bounds for parity-oblivious random access codes**. *New Journal of Physics* 18(4), p. 045003, doi: 10.1088/1367-2630/18/4/045003.

[7] Nicolas Delfosse, Philippe Allard Guerin, Jacob Bian & Robert Raussendorf (2015): **Wigner function negativity and contextuality in quantum computation on rebits**. *Physical Review X* 5(2), p. 021003, doi:10.1103/PhysRevX.5.021003.

[8] Máté Farkas & Jedrzej Kaniewski (2019): **Self-testing mutually unbiased bases in the prepare-and-measure scenario**. *Physical Review A* 99(3), p. 032316, doi: 10.1103/PhysRevA.99.032316.

[9] Ernesto F Galvao (2005): **Discrete Wigner functions and quantum computational speedup**. *Physical Review A* 71(4), p. 042302, doi: 10.1103/PhysRevA.71.042302.

[10] Andrzej Grudka, Karol Horodecki, Michał Horodecki, Waldemar Kłobus & Marcin Pawłowski (2014): **When are Popescu-Rohrlich boxes and random access codes equivalent?** *Physical review letters* 113(10), p. 100401, doi: 10.1103/PhysRevLett.113.100401.

[11] Alley Hameedi, Armin Tavakoli, Breno Marques & Mohamed Bourennane (2017): **Communication games reveal preparation contextuality**. *Physical review letters* 119(22), p. 220402, doi: 10.1103/PhysRevLett.119.220402.

[12] Mark Howard, Joel Wallman, Victor Veitch & Joseph Emerson (2014): **Contextuality supplies the 'magic' for quantum computation**. *Nature* 510(7505), pp. 351–355, doi:10.1038/nature13460.

[13] Shane Mansfield & Elham Kashefi (2018): **Quantum advantage from sequential-transformation contextuality**. *Physical review letters* 121(23), p. 230401, doi: 10.1103/PhysRevLett.121.230401.

[14] Hakop Pashayan, Joel J Wallman & Stephen D Bartlett (2015): **Estimating outcome probabilities of quantum circuits using quasiprobabilities**. *Physical review letters* 115(7), p. 070501, doi:10.1103/PhysRevLett.115.070501.

[15] Marcin Pawłowski, Tomasz Paterek, Dagomir Kaszlikowski, Valerio Scarani, Andreas Winter & Marek Żukowski (2009): **Information causality as a physical principle**. *Nature* 461(7267), pp. 1101–1104, doi:10.1038/nature08400.

[16] Debashis Saha, Paweł Horodecki & Marcin Pawłowski (2019): **State independent contextuality advances one-way communication**. *New Journal of Physics* 21(9), p. 093057, doi: 10.1088/1367-2630/ab4149.

[17] Robert W Spekkens (2005): **Contextuality for preparations, transformations, and unsharp measurements**. *Physical Review A* 71(5), p. 052108, doi:10.1103/PhysRevA.71.052108.

[18] Robert W Spekkens, Daniel H Buzacott, Anthony J Keehn, Ben Toner & Geoff J Pryde (2009): **Preparation contextuality powers parity-oblivious multiplexing**. *Physical review letters* 102(1), p. 010401, doi:10.1103/PhysRevLett.102.010401.

[19] Armin Tavakoli, Alley Hameedi, Breno Marques & Mohamed Bourennane (2015): **Quantum random access codes using single d-level systems**. *Physical review letters* 114(17), p. 170502, doi:10.1103/PhysRevLett.114.170502.

[20] Victor Veitch, Christopher Ferrie, David Gross & Joseph Emerson (2012): **Negative quasi-probability as a resource for quantum computation**. *New Journal of Physics* 14(11), p. 113011, doi:10.1088/1367-2630/14/11/113011.