Data re-uploading for a universal quantum classifier
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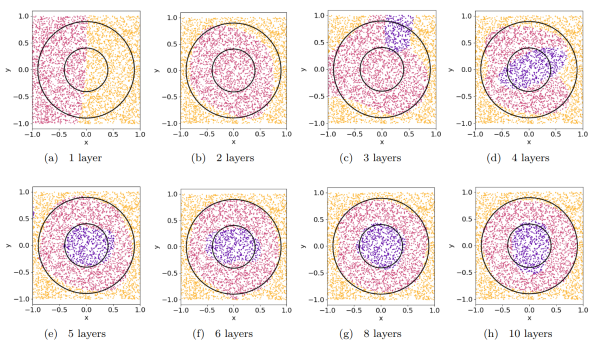
A single qubit provides sufficient computational capabilities to construct a universal quantum classifier when assisted with a classical subroutine. This fact may be surprising since a single qubit only offers a simple superposition of two states and single-qubit gates only make a rotation in the Bloch sphere. The key ingredient to circumvent these limitations is to allow for multiple data re-uploading. A quantum circuit can then be organized as a series of data re-uploading and single-qubit processing units. Furthermore, both data reuploading and measurements can accommodate multiple dimensions in the input and several categories in the output, to conform to a universal quantum classifier. The extension of this idea to several qubits enhances the efficiency of the strategy as entanglement expands the superpositions carried along with the classification. Extensive benchmarking on different examples of the single- and multi-qubit quantum classifier validates its ability to describe and classify complex data.

The basic structure of a single-qubit classifier is the following. Data points are re-uploaded multiple times along the circuit by using rotational gates. Thus, each layer of this classifier model combines encoding and processing with a single operation $L(i) = U\left(\vec{\theta}_i + \vec{w}_i \circ \vec{x}\right)$, where $\vec{w}_i \circ \vec{x} = \left(w_i^1 x^1, w_i^2 x^2, w_i^3 x^3\right)$, $\vec{x}$ are the data points and $\vec{w}$ and $\vec{\theta}$ the parameters to be optimized. It can be shown, that this structure allows to prove the universality of this quantum classifier by means of the Universal Approximation Theorem.

Next, we assign to each class a pure state on the Bloch sphere and train the quantum circuit classifier to find the $\vec{\theta}_i$ and $\vec{w}_i$ that best approximate the training data points to their corresponding target states. Inspired by the theorems of perfect discrimination measurements, we select the most orthogonal states on the Bloch sphere, e.g. the $|0\rangle$ and $|1\rangle$ states for a binary classification, the vertices of a regular tetrahedron for a four-class classification, etc. As a cost function, we can simply use the fidelity of the single-qubit classifier output state with respect to the target state or a more sophisticated cost function that weights the error made by the classifier.

We test this quantum classifier model with different data sets. One of those is a two dimensional three-class classification: points inside, outside and contained in an annulus. The results for this particular problem are plotted in the figure below.



(a) 1 layer (b) 2 layers (c) 3 layers (d) 4 layers
(e) 5 layers (f) 6 layers (g) 8 layers (h) 10 layers

Finally, we explore the multi-qubit quantum classifier with data re-uploading, that is, defining each layer with the single-qubit rotations as the one described above, and entangling gates between the qubits. We compare its performance with the single-qubit classifier obtaining better results for the data sets studied.

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[1] *Data re-uploading for a universal quantum classifier*, A. Pérez-Salinas, A. Cervera-Lierta, E. Gil-Fuster and J. I. Latorre, Quantum **4**, 226 (2020), arXiv:1907.02085 [quant-ph].