Quantum unary approach to option pricing

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Abstract. We present a quantum algorithm for European option pricing in finance, where the key idea is to work in the unary representation of the asset value. The algorithm needs novel circuitry, and amplitude amplification is used to gain quantum advantage. Unary representation simplifies the circuit and provides a post-selection that allows for a substantial error mitigation. However, unary representation offers lower resolution given a number of qubits. We compare the performance of both option pricing algorithms using error maps, and find that unary representation may bring an advantage in practise.

Keywords: quantum, finance, amplitude estimation, error mitigation, NISQ

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1 Introduction

Quantum computing provides new strategies to address problems that nowadays are considered difficult to solve by classical means. The first quantum algorithms showing a theoretical advantage of quantum computing are well-known since the 1990s [1], [2]. Nevertheless, current quantum devices are yet not powerful enough to run quantum algorithms competing against state-of-the-art classical algorithms, *i, e,* they are in their NISQ stage.

Quantitative finance is expected to be transformed by quantum devices. Notably, pricing of derivatives is a problem where many computational obstacles are suited to be overcome via QC. This problem is usually solved via the Black-Scholes model [3] and Monte Carlo simulations.

In the following, we propose a novel quantum algorithm for option pricing. The key new idea is to construct a quantum circuit that works in the unary basis of the asset value. This algoritm is composed of an amplitude distributor module, a payoff calculator and an amplitude estimation step. The unary approach provides a substantial simplification of the quantum circuit, and a post-selection subroutine that allows for error mitigation.

2 Unary algorithm

The main feature of this algorithm is the unary bases it is designed to work on. The unary basis is defined as the subspace of the Hilbert space with only one qubit in the |1> state. Given a fixed number of qubits, the unary scheme allows for a lower precision than the binary one. Indeed, only n out of the 2^n basis elements of the Hilbert space are used.

The algorithm consists in three different pieces. The first one is the amplitude distributor, that builds a quantum state whose amplitudes in the unary basis follow the BS model for a given asset. This component can be done exactly, with no variational steps. The second piece is a payoff calculator, which is made using several controlled Ry gates. The amplitude estimation is performed to obtain quantum advantage. The design of the quantum algorithm follow the same steps as previous works [4].

All pieces simplify in the unary basis with respect to the standard binary basis, resulting in a circuit with two main advantages: 1) the algorithm is much shorter, and 2) the connectivity requested for performing the algorithm with no extra steps is simpler. Thus, there a regime for small precision, in which the number of gates required for the unary algorithm is smaller.

In addition, a post-selection scheme exists in the unary case. The unary algorithm ideally works within the unary subspace. Thus, any outcome that does not fulfill this requirement is discarded.

Amplitude Estimation cannot be performed in its original formulation, but following an iterative procedure, based on [5].

3 Results

Simulations for this algorithm and the binary counterpart were performed for 8 and 3 qubits, respectively, in order to compare like with like. Depolarizing and measurement noise was added to the simulation. Simulations show that both the unary and binary algorithm preform similarly when no amplitude estimation is applied, that is, when there is no quantum advantage. However, when amplitude estimation is considered, the unary algorithm presents much more resilience against errors, see Fig. 1.

4 Conclusions

We have illustrated our algorithm in the particular case of a single European option. Unary representation definitely offers relevant advantages over the binary one. The algorithm is simpler, and a consistency check is provided. The ability to post-select faithful runs mitigates errors and increases the performance of the quantum algorithm. In addition, Amplitude Estimation may be performed successfully only in the unary basis, considering error levels in NISQ devices, since the procedure is more resilient to errors than the binary one.

References

[1] Peter W. Shor, Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer, 1999
[2] Lov K. Grover, A fast quantum mechanical algorithm for database search, 1996
[3] Fischer Black and Myron Scholes, The Pricing of Options and Corporate Liabilities, 1973
[4] Nikitas Stamatopoulos, Daniel J. Egger, Yue Sun, Christa Zoufal, Raban Iten, Ning Shen and Stefan Woerner, Option Pricing using Quantum Computers, 2020
[5] Yohichi Suzuki, Shumpei Uno, Rudy Raymond, Tomoki Tanaka, Tamiya Onodera and Naoki Yamamoto, Amplitude estimation without phase estimation, 2020

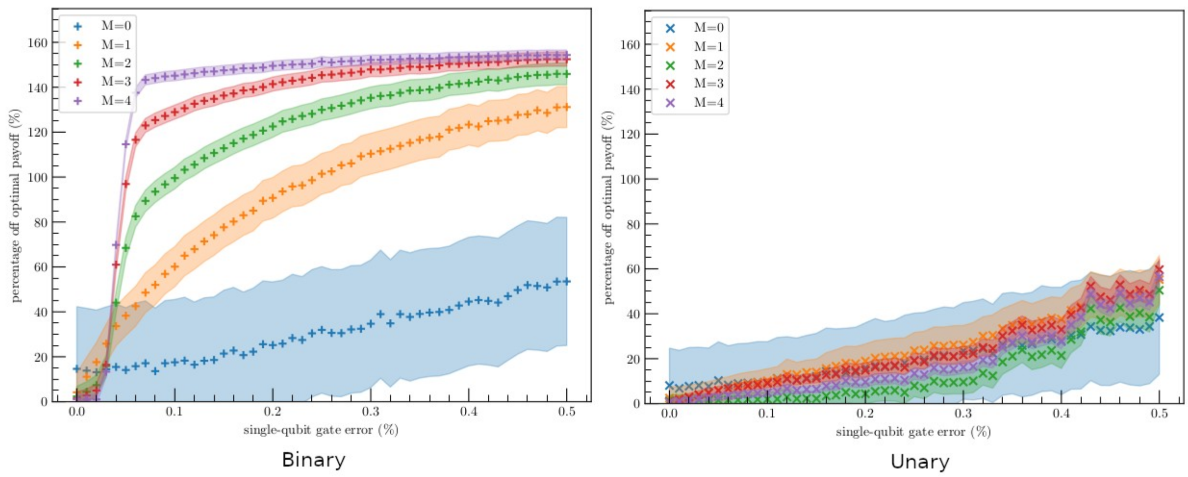

Figure 1: Results for the simulations for the binary and the unary algorithms for 3 and 8 qubits respectively. The y axis stands for the error obtained in the payoff, while the x axis represents the single-qubit gate error for the model. Each color depicts an iteration of amplitude estimation. It is clear that when no amplitude estimation is applied (blue), performances of both methods are similar. On the contrary, when it is applied, the unary algorithm presents a more resilient behavior. The saturation regime of the binary algorithm corresponds to the random circuit.