Nonclassical steering and the Gaussian steering triangoloids

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Abstract. We fully characterize the mechanism by which nonclassicality according to the Glauber P-function can be conditionally generated on one mode of a two-mode Gaussian quantum state by generic Gaussian measurements on the other mode. For two-mode squeezed thermal states, we visualize the whole set of conditional states constructing Gaussian steering triangoloids and we show that nonclassicality can be induced in this way if and only if the initial state is EPR-steerable. In the more general case, we recognize two types of quantum correlations: weak and strong nonclassical steering, the former being independent of entanglement, and the latter implying EPR steerability.

Keywords: quantum correlations, steering, entanglement, nonclassicality

Nonclassical quantum states are among the most powerful tools in modern technology, where they have been employed for metrology [1], quantum teleportation [2], quantum key distribution (QKD) [3] and quantum computing [4]. For continuous-variable (CV) quantum states, the definition of nonclassicality according to the singularity or negativity of the Glauber P-function is the most physically motivated, having relevant implications in quantum optics [5,6]. For a system of two bosonic modes described by Gaussian quantum states, we explore how P-nonclassicality may be conditionally generated or influenced on one mode by Gaussian measurements on the other mode. We begin the exploration with the experimentally relevant and theoretically vast class of two-mode squeezed thermal states (TMST), arriving at the notion of *nonclassical steering* (NS) through the graphical tool of Gaussian steering triangoloids that allow to visualize all conditional states of one mode that can be prepared by Gaussian measurements on the other mode and classical communication of the outcome. We then derive a necessary and sufficient condition for any TMST to be nonclassically steerable, proving that entanglement is only necessary for such a process. We then apply this result to the noisy propagation of a twin-beam state, where just one mode directly interacts with a Markovian and purely thermal environment. We evaluate the time after which NS is no longer achievable and we compare it with the disentangling time. We go on to generalize the notion of NS to a generic Gaussian state of two modes, recognizing two distinct types of quantum correlations: *weak nonclassical steering*, being independent of initial entanglement, and *strong nonclassical steering*, implying EPR-steerability, therefore also entanglement. We show that the two concepts coincide for TMSTs, and moreover they merge with the notion of EPR steering [7,8], a fact that may find applications to one-sided device-independent QKD and that highlights a novel, unexpected link between P-nonclassicality and entanglement. Finally, we discuss the set-theoretical relations between the new quantum correlations we described, and we compare them with Reid’s criterion for the EPR paradox [9,10] and with a recently discovered condition for remote Wigner negativity generation by one-photon subtraction [11,12].

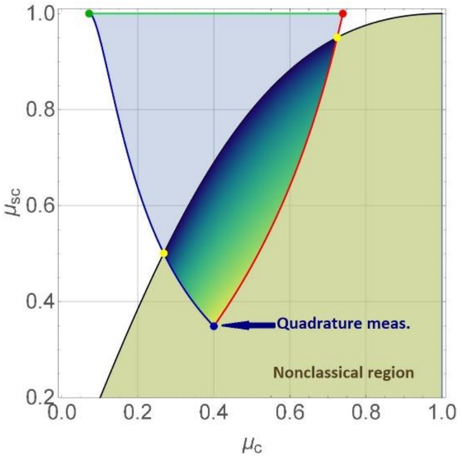

Figure 1 Gaussian steering triangoloid for a nonclassically steerable TMST states. Points inside the triangoloid represent all conditional states achievable by Gaussian measurements on the other mode. The shaded region contains all *nonclassical* conditional states. The shading reflects the degree of nonclassicality, captured by the *nonclassical depth.*

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