Creating ensembles of dual unitary and maximally entangling quantum evolutions

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Abstract

Entanglement of unitary operators quantified in terms of operator entanglement, and average entanglement created by the unitary evolution quantified as entangling power, has paramount importance in quantum information theory as well as in many-body physics. The unitaries that maximize operator entanglement, called dual unitaries, can be used as an ingredient for analytically solvable nonintegrable many-body systems. Thus it is an open question as to how to construct dual unitaries in arbitrary dimensions. In this work we provide an efficient iterative method to generate an ensemble of such dual unitaries having maximal operator entanglement and entangling power acting on bipartite quantum systems.

Unentangled states are often entangled due to the action of entangling unitary operators in the circuit paradigm of quantum computing [1]. Thus how entangled unitary operators themselves are (measured by *operator entanglement*) [2], and how much entanglement they can produce, on the average, acting on unentangled states (measured by *entangling power*) [3] are of primary interest. They have also started forming a means to characterize complexity in many-body systems [4, 5, 6] and the study of quantum chaos [7, 8].

Maximally entangled bipartite states such as the prototypical Bell states of two qubits, and its generalization to any dimension: $\sum_{i=1}^{d} |i_A i_B\rangle \sqrt{d}$ (where $|i_{A,B}\rangle$ form a complete orthonormal basis in each of the particle spaces) is straightforward to construct and characterize. On the contrary, while it is easy to state conditions under which an operator may have maximal operator entanglement or entangling power, it seems surprisingly difficult to construct their Bell state equivalents, let alone to characterize and parameterize them [9].

Further motivation for constructing maximally entangled unitary operators comes from recent observations concerning lattice models wherein a space-time duality allows for some analytical results, even for nonintegrable systems [10, 11, 12, 13]. More explicitly, using a "dual unitary" [11], as the nearest neighbor interaction in many-body systems, leads to solvable correlation functions.

In this work, we outline a protocol that leads iteratively to a systematic increase of the operator entanglement, leading to operators that are arbitrarily close to being dual unitaries. A subset of these could also be arbitrarily close to 2-unitaries , but we outline an alternative strategy that, while not monotonic, leads to near 2-unitaries. In particular, for the case of local dimension $d = 3$ (qutrits) and to some extent $d = 4$, we show that the procedure leads to a considerable measure of 2-unitaries. It may be noted that for $d = 2$, the qubit case, 2-unitaries do not exist [14, 2]. Starting from random unitary matrices selected uniformly from the group $U(d^2)$, the circular unitary ensemble or CUE of random matrix theory (RMT) [15, 16], these protocols generate an ensemble of dual unitaries for all local dimensions and an ensemble of 2-unitaries for $d = 3$ and $d = 4$.

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