# Spatial entanglement between two walkers with symmetric and anti-symmetric coins undergoing discrete-time quantum walk

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**Abstract.** We study the dynamics of the spatial entanglement between two initially independent walker undergoing discrete quantum random walk in one dimension according to different initial coin states. The result confirms that the walkers will become entangled as long as the coins are initially entangled. We also show that the degree of the entanglement will increase when we perform post-selection based on the outcome of the coins rather than just tracing all the coins out. However, in both cases, the entanglement will start to fluctuate after that in a similar fashion to the dynamics of a damping oscillator, unless we start with certain initial states. Our result shows that the entanglement will not change only when the states of the particles are effectively one of the Bell states. We also show that the exchange symmetry is preserved throughout the quantum walks.

**Keywords**: spatial entanglement, quantum random walk, damping oscillator, exchange symmetry

## 1 Introduction

Two-particle quantum walks have been studied in several different settings, for example, in [1,2,3,4,5,6]. Here, we study another aspect of the walk which is the dynamics of spatial entanglement between two walkers with symmetric/anti-symmetric coins and also the role of post-selection on the coin states. We define the coin and shift operator in the same way as [1].

## 2 Results

We are particularly interested in the entanglement between the two walkers when the two coins are post-selected depending on whether they have the same/different spins. Figure 1 shows that the entanglement bahaves in a similar way to a damping oscillator. To explain this behaviour, we study the dynamics of entanglement between two walkers with the anti-symmetric coin state (ACS) and symmetric coin state (SCS). We found that for ACS the walkers state after post-selection is always in the form,

$$|x, y> - |y, x>.$$

This is always a Bell state. Thus the entanglement is contant in this case. The walkers state for SCS after post-selection is always in the form,

$$|u, v> + |v, u>.$$

This will be a Bell state iff <u,v|v,u>=0 and the entanglement is again constant. If <u,v|v,u>≠0, the state is not Bell and we find the entanglement is oscillating.

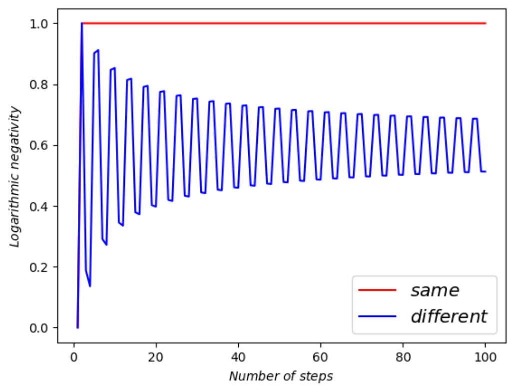


**Figure 1** Spatial Entanglement between two walkers after post-selection VS walking step for SCS.

## 3 Conclusion

We study ACS and SCS and discover that for ACS the walkers state after post-selection is always anti-symmetric and the entanglement is constant, while the walkers state for SCS is symmetric and the entanglement is constant if it's Bell, if not the entanglement oscillates.

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