Classical communication through quantum causal structures

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Abstract. Quantum mechanics allows operations to be in indefinite causal order. Recently there have been active discussions on enhanced communication strategies through exotic causal structures. In light of this, through the process matrix formalism, we formulate different classical capacities for a bi-partite quantum process. We find that a one-way communication protocol through an arbitrary process cannot outperform a causally separable process, i.e., we can send at most one bit per qubit. Next, we study bi-directional communication through a causally separable process. Our result shows, a bi-directional protocol cannot exceed the limit of one way communication protocol. Finally, we generalise this result to multi-party broadcast communication protocol through a definite ordered process.

Keywords: Quantum causal structure, Quantum Shannon theory.

An important question in quantum information is the rate at which a quantum channel can transmit classical information, as quantified by the *classical capacity* of the channel [1,2]. Holevo's seminal result [3] and following works [4,5], provide upper bounds on the classical capacity, showing that each qubit can communicate at most one bit of classical information.

In a typical quantum communication protocol, the parties act in a fixed order. However, more general situations are possible, where the possibilities of *indefinite causal structure* exist, i.e., scenarios where the lack of order between the parties cannot be reduced to classical ignorance [6,7].

In light of the foundational and applied relevance, it is important to understand how general quantum causal structure affects classical communication. In particular, one may wonder whether an indefinite causal structure can augment the classical communication capacity and possibly exceed the Holevo bound [3].

We address this gap through the process matrix formalism [7]. We develop expressions for the asymptotic capacity of a process, under different encoding and decoding settings, reducing to analogue expressions for quantum channels. We explore one-way communication protocols through an arbitrary process and show that such scenarios cannot exceed communication in definite causal order, i.e., we can send at most one bit per exchanged qubit. We also explore two-party communication protocols when causal order is definite but unknown (probabilistic). In such situations, the total bi-directional communication cannot exceed the maximum one-way communication, again, at most one bit per qubit in either direction. This extends to a similar bound for communication between multiple parties in a definite (but possibly probabilistic) causal order [8].

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