Fault-tolerant quantum speedup from constant depth quantum circuits

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Extended abstract−A critical milestone in the field of quantum computing is the near-term demonstration of a so-called *quantum computational advantage*. In principle, this could be done by constructing a quantum device which provably outperforms its classical counterpart for a specific computational task. Among the promising candidates for demonstrating a near-term quantum advantage are the so-called *sub-universal* quantum devices which are not universal, in the sense that they cannot perform any quantum computation, but are realizable in principle by our current technologies [1]. Several examples of such practically motivated sub-universal models which nevertheless capture a sense of quantum advantage have been discovered in recent years. In most of these works, sampling from the output probability distribution of these sub-universal devices has been shown to be classically impossible to do efficiently, provided widely believed complexity theoretic conjectures hold [2, 3, 4, 5]. Thus, these devices demonstrate what is known as an exponential *quantum speedup* (sometimes referred to as quantum supremacy) [6].

The first experimental demonstration of quantum speedup is a major milestone in quantum information. Recent audacious experimental efforts [7] and subsequent proposals of their classical simulation [8] bring to light the challenges and subtleties of achieving this goal. Among these challenges is the important issue of dealing with noise in the quantum device. Indeed, it was shown in multiple works [9, 10, 11, 12, 13, 14] that noise can very easily lead to the breakdown of quantum speedup, rendering the output probabilities of these devices (which in the noiseless case demonstrate quantum speedup) classically simulable efficiently. There is clearly a great need to understand better the effect of noise, and develop methods of mitigation. Unfortunately, introducing full-blown fault-tolerant techniques to deal with the noise [15, 16, 17] adds a significant overhead, in terms of ancilla qubit number, circuit depth, and number of interactions with a classical device. All of these typically scale with the system size. This is undesirable as it usually takes us out of the simplified computational models which made sub-universal devices attractive in the first place. Though there are exceptions to this for specific sub-universal models and specific (yet slightly unrealistic) types of noise [9].

In this work, we study how quantum speedup can be demonstrated in the presence of noise for a family of sampling problems. We take as our noise model the local stochastic quantum noise, commonly studied in the quantum error correction and fault-tolerance literature [18, 19, 20, 21]. Our sampling problems are built on a family of schemes essentially based on local measurements on regular graph states composed of n qubits, which correspond to constant depth 2D nearest neighbor (NN) quantum circuits showing quantum speedup [22, 23, 24, 25]. We show that these can be made fault-tolerant in a way which maintains constant depth of the quantum circuits, albeit with $poly(n)$ overhead in the number of ancilla qubits used, and at most two rounds of efficient classical computation during the running of the circuit.

We present two constructions, each composed of a polynomial number of noisy qubits, some of which are prepared in noisy magic states [26]. The first of our constructions is a constant depth quantum circuit composed of single and two-qubit NN Clifford gates in four dimensions. This circuit has one layer of interaction with a classical computer before final measurements. Our second construction is a constant depth quantum circuit with single and two-qubit NN Clifford gates in three dimensions, but with two layers of interaction with a classical computer before the final measurements.

In constructing our circuits, we use various concepts from [18] such as single shot fault-tolerant logical state preparation and forward propagation of local stochastic noise through a Clifford circuit. We also develop various new techniques, which could have potential applications outside of this work, such as constant depth non-adaptive magic state distillation [26] and constant depth output routing. Our developped techniques can be understood naturally when looking at the framework of measurement based quantum computation (MBQC) [27].

Closest to our work are those of [18] and [28]. In [18], a constant depth 3D NN Clifford circuit is constructed to perform a task which cannot be performed by any *constant* depth classical circuit. Their result is unconditional, robust to local stochastic noise, and their circuit does not have any interaction with a classical device during running [18]. In our case, we show that constant depth 3D NN Clifford circuits, with non-Clifford inputs, can perform a task which no *polynomial*

depth classical circuit can perform. However, our result is conditioned on two complexity theoretic conjectures holding, is robust to local stochastic noise, and requires two interactions with a classical computer while running. In [28] it is shown that constant depth 3D NN Clifford circuits with non-Clifford input can perform a task which no polynomial time classical circuit can perform. These circuits are robust to a specific noise [29], and have no interaction with a classical device when running. However, the main disadvantage of the construction in [28] is that it is impractical, in the sense that one should repeat the experiment (construct the circuit, then measure) an exponential number of times in order to observe an instance which is hard for the classical computer to simulate. In our work, we overcome this problem using our new distillation technique, thereby making the appearance of a hard instance very likely in only a few repetitions of the experiment, a feature called single-instance hardness [22].

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