An unbounded number of independent observers can share the nonlocality of a single maximally entangled qubit pair*

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Quantum theory predicts the possibility of measuring correlations that cannot be explained by standard notions of causality [1]. These 'nonlocal' correlations are a crucial resource for device-independent tasks such as key-distribution [2–4] and randomness expansion [5–7]. The idea is that because such correlations cannot be explained using local hidden variables, the individual outcomes are random and unpredictable to an adversary [3, 8, 9]. Several recent advances have enabled the first experimental demonstrations of device-independence [10–12]. However, further theoretical and experimental advances are needed before this becomes a practical technology.

In this work we study fundamental limits on nonlocality, asking whether a single pair of entangled qubits could generate a long sequence of non-local correlations. Such an approach could be useful for situations where a significant bottleneck lies in the state generation, such as in nitrogen vacancy based experiments [13]. We study a scenario in which a single Alice tries to establish nonlocal correlations with a sequence of Bobs who measure sequentially one half of an entangled qubit pair. An additional restriction we impose is that each Bob in the sequence can only send a single qubit (his post-measurement state) to the next. In particular, the classical information pertaining to measurement choices and outcomes of each Bob is not shared. It is in this sense that the Bobs act independently of one another.

This sequential scenario (see Fig. 1) was introduced in [14]. There it was shown that by modifying the input distributions of the Bobs, so that one of the inputs is highly favoured, an unbounded number of Bobs could each have an expected CHSH violation [15] with the single Alice who measures once. However, the authors also mentioned numerical evidence suggesting that if the input distributions were not modified (each Bob chooses a binary input uniformly at random) then at most two Bobs would be able to have an expected CHSH violation with Alice.

By constructing an explicit measurement strategy, we show that, contrary to what was previously thought, there is no bound on the number of independent Bobs (with uniform inputs) that can have an expected violation of the CHSH inequality with Alice. We exhibit a class of initial two-qubit entangled states that are capable of achieving an unbounded number of violations, which includes all pure two-qubit entangled states.

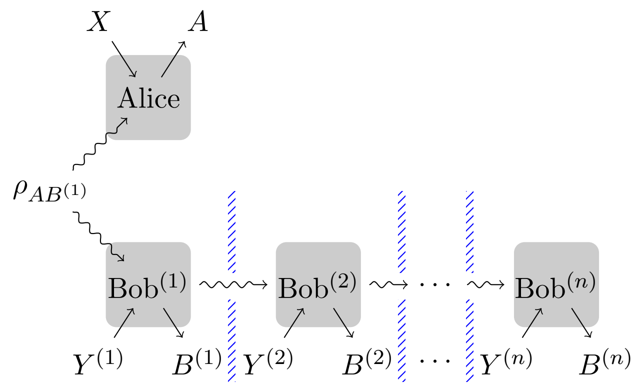

Figure 1: A schematic of the considered sequential CHSH scenario. All random variables $X, A, Y^{(i)}, B^{(i)}$ for $i = 1, \ldots, n$ have binary outcomes. A quantum state $\rho_{AB^{(1)}}$ is initially shared between Alice and Bob$^{(1)}$. After Bob$^{(1)}$ has performed his randomly selected measurement and recorded the outcome he passes the qubit post-measurement state to Bob$^{(2)}$ who repeats this process. Only the qubit post-measurement states are sent to the next Bob.

Our result sheds a new light on the analysis of sequential scenarios. Previous works [14,16–21], which analyze nonlocality, steering and entanglement in the sequential setting, have restricted their analyses to measurement strategies where the sharpness of the measurements that each Bob uses are equal. With this restricted class of measurement strategies, strong limitations on the possible performance of the various tasks were found. In particular, the work of [17] shows that with the restricted class of measurement strategies at most two Bobs can violate *any* 2-outcome Bell inequality with Alice in this scenario when starting with a maximally entangled state. In the present work we show that these limitations can be overcome by using measurements with unequal sharpness. This overturns some previous results in the area and highlights the importance of considering the most general measurement strategies.

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