Signatures of avoided energy-level crossings in tomographic entanglement indicators

B. Sharmila[1], S. Lakshmibala[1] and V. Balakrishnan[1]

[1] *Department of Physics, Indian Institute of Technology Madras, Chennai 600036, India.*

Abstract. Investigations on multipartite systems close to avoided energy-level crossings reveal interesting features such as the extremisation of entanglement. Conventionally, the estimation of entanglement directly from experimental observations involves either one of two approaches: Identifying linear correlations between relevant observables, or rigorous but error-prone quantum state reconstruction from experimentally-obtained tomograms. In contrast, we describe entanglement indicators that can be obtained directly from tomograms (**arXiv:2006.13536**), circumventing elaborate state reconstruction, in both continuous-variable and hybrid quantum systems. These indicators capture nonlinear correlations between subsystems. We establish that these indicators are in excellent agreement with the subsystem von Neumann entropy.

Keywords: Energy-level crossings, Entanglement, Tomograms, Tomographic entanglement indicator.

Measurement of any observable in a quantum mechanical system yields a histogram of the state of the system in the basis of that observable. By a judicious choice of a quorum of observables, a set of histograms (the tomogram) can be experimentally obtained. The density matrix is reconstructed from the tomogram, and entanglement indicators such as the subsystem von Neumann entropy $\xi_{\text{SVNE}} = -\text{Tr}\,(\rho \log_2 \rho)$, where ρ is the density matrix of either one of the subsystems of a bipartite system, is calculated. In contrast to spin systems where the quorum is only a finite set of observables, in continuous-variable (CV) systems, in principle, an infinite set of histograms constitute the tomogram. However, in practice, only a finite set of histograms can be obtained, and the challenge lies in identifying efficient entanglement indicators that capture nonlinear correlations between subsystem observables *directly* from tomograms. We have carried out this analysis in CV systems such as the double-well Bose-Einstein condensate [1], and a bipartite system comprising a multi-level atom interacting with a radiation field [2]. We have extended our investigations to include a multipartite hybrid quantum (HQ) system [3] modelled by the Tavis-Cummings Hamiltonian [4].

The focus is on avoided energy-level crossings in these systems, as earlier literature [5–7] indicates that entanglement indicators extremise at avoided crossings. These avoided crossings are mirrored in the spacings between the energy levels. The latter change significantly with change in the system parameters, with two or more levels moving close to each other for specific values of the parameters, and then moving away as these values change. This feature is called avoided energy-level crossing. Typically, this depends on the strengths of the nonlinearity and the couplings between subsystems.

We have identified several entanglement indicators directly from relevant tomograms. These include ξ_{TEI} based on mutual information [8] and ξ_{IPR} based on inverse participation ratios [9]. (The participation ratio is a measure of delocalization in a given basis). In addition, we have assessed some indicators familiar in classical tomography. While these are well known quantifiers of classical correlation, their description does not preclude their application in the quantum regime. Such indicators are ξ_{BD} based on the Bhattacharyya distance between classical statistical ditributions [10] and ξ_{PCC} based on the Pearson correlation coefficient [11] which captures only linear correlation.

Our results are both useful and novel since the procedures used circumvent detailed state reconstruction and reveal a set of indicators that compare well with ξ_{SVNE}.

References

[1] L. Sanz, M. H. Y. Moussa, and K. Furuya. *Ann. Phys. (N.Y.)*, 321:1206, 2006.

[2] G. S. Agarwal and R. R. Puri. *Phys. Rev. A*, 39:2969, 1989.

[3] D. S. Shapiro, P. Macha, A. N. Rubtsov, and A. V. Ustinov. *Photonics*, 2:449, 2015.

[4] M. Tavis and F. W. Cummings. *Phys. Rev.*, 170:379, 1968.

[5] G. Vidal, J. I. Latorre, E. Rico, and A. Kitaev. *Phys. Rev. Lett.*, 90:227902, 2003.

[6] J. Karthik, A. Sharma, and A. Lakshminarayan. *Phys. Rev. A*, 75:022304, 2007.

[7] S. Oh, Z. Huang, U. Peskin, and S. Kais. *Phys. Rev. A*, 78:062106, 2008.

[8] B. Sharmila, K. Saumitran, S. Lakshmibala, and V. Balakrishnan. *J. Phys. B: At. Mol. Opt.*, 50:045501, 2017.

[9] W. G. Brown, L. F. Santos, D. J. Starling, and L. Viola. *Phys. Rev. E*, 77:021106, 2008; B. Sharmila, S. Lakshmibala, and V. Balakrishnan. *Quantum Inf. Process.*, 18:236, 2019.

[10] T. Kailath. *IEEE T. Commun. Techn.*, 15:52, 1967.

[11] R. Smith. *Stat*, 4:291, 2015.