An energetic perspective on rapid quenches in quantum annealing

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Abstract. Well-developed theoretical tools exist to analyse how quantum dynamics can solve computational problems by varying Hamiltonians slowly (adiabatically). However, relatively few tools exist for the opposite limit of rapid quenches, used in quantum annealing and quantum walks. We develop a theoretical understanding and several practical tools for this regime. By analysing various energy expectation values, we show that monotonic quenches will yield a better result on average than random guessing. By characterising local dynamics, we identify cases where rapid quenches will lead to a substantially improved numerical scaling. We then use these tools to develop heuristics for choosing control parameters.

Keywords: adiabatic, quantum walk, continuous-time

Adiabatic quantum computing (AQC) [1], in which a Hamiltonian is varied slowly, is backed by well-developed theoretical tools. Likewise, continuous-time quantum walks (CTQW) [2], in which the Hamiltonian is held constant, and their application to quantum search are well understood analytically [3].

In [4], by applying CTQW to finding groundstates of spin-glasses, it was shown that using more physically feasible problem mappings (the Ising model) than unstructured search leads to drastically different behaviour and improved solution probability. Promisingly, using many short repeats hugely outperforms a single long run, making it more practical for near term hardware, on which maintaining long coherence times is difficult. Unless P=NP, algorithms which succeed with O(1) probability must maintain coherence exponentially long, while many repeats of algorithms with exponentially decreasing success probability can have mildly scaling coherence time. Furthermore, via numerical comparison with less structured problems, such as the random energy model [5], the salient aspects of the structure were identified. The use of CTQW for optimisation has been independently explored in [6].

In [7], it was shown that AOC and CTOW sit at the extreme ends of a spectrum of continuous time quantum computing methods, including the rapid quenches often used in quantum annealing [8, 9]. In [10], we develop several theoretical tools for the rapid quench regime. First, we analyse the energy expectation value of different elements of the Hamiltonian. From

this, we show that monotonic quenches, where the strength of the problem Hamiltonian is consistently increased relative to fluctuation (driver) terms, will yield more optimal solutions on average than random guessing. Secondly, we develop methods to determine whether dynamics will occur locally under rapid quench Hamiltonians, and identify cases where a rapid quench will lead to a substantially increased solution probability. We also show how these tools can provide efficient heuristic estimates for control parameters (e.g Fig. 1), a key requirement for practical application of quantum annealing.

The talk will be based on [4] and [10] and will begin with a gentle introduction to continuous-time quantum computing, with a focus on physically realistic problem mappings. I will then cover the theoretical understanding and tools we have developed for CTQW and rapid quenches, and finally discuss examples of using the heuristic methods these tools provide.

Figure 1 Average success probability for a quantum walk run for a short time to find the ground-state of a spin-glass instance (with n spins) using the problem specific heuristic hopping rate γ_{heur} from [4], compared to the more general heuristic γ_{Dyn} from [10].

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