

Positive maps and Matrix Contractions from the Symmetric Group

Felix Huber

*ICFO - The Institute of Photonic Sciences, 08860 Castelldefels (Barcelona), Spain**

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Using techniques from quantum information theory, we develop a systematic method to obtain operator inequalities and identities in several matrix variables. These take the form of polynomial-like expressions that involve matrix monomials $X_{\alpha_1} \cdots X_{\alpha_r}$, their traces $\text{tr}(X_{\alpha_1} \cdots X_{\alpha_r})$, and tensor products. As a result, we obtain new operator matrix inequalities for the positive cone, characterize the set of multilinear equivariant positive maps, and construct matrix identities on tensor product spaces. This unifies several concepts from quantum information theory as found in the study of quantum channels, entanglement detection, quantum codes, and monogamy of entanglement.

Object of study — We study matrix contractions [1]: expressions that can be realised as linear combination of matrix monomials (including $\mathbb{1}$), their traces, and their products. E.g.

$$ABC + \text{tr}(B)CA - 2 \text{tr}(AC) \text{tr}(B)\mathbb{1}. \quad (1)$$

We demand the expressions to be multilinear and equivariant, which means that they are of degree one in each variable and composed of matrix monomials and their traces. We ask: which multilinear matrix contractions are positive on the positive cone (e.g. $A, B, C \geq 0$)? Which expressions vanish on all $d \times d$ matrices?

Results — We characterize all such much matrix inequalities for the positive cone (i.e. multilinear equivariant positive maps) and obtain a one-to-one correspondence with Werner state witnesses. We also characterize the set of all polynomial identities with tensor product structure. This unifies and extends certain techniques that appear in the study of quantum channels [2], entanglement [3], quantum marginal problem [4–6], monogamy [6], quantum codes [7], joint measurability [8], time reversal [9]; it also expands on results and introduces new concepts in the field of polynomial identity rings [10–12].

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* felix.huber@icfo.eu

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