A variational toolbox for quantum multi-parameter estimation

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Abstract. We propose a variational quantum algorithms feasible on NISQ devices that can address a challenge central to the field of quantum metrology: The identification of near-optimal probes and measurement operators for noisy multi-parameter estimation problems. Our framework is applicable to both discrete and continuous variable settings on different physical platforms. We demonstrate the practical functioning of the approach through numerical simulations, showcasing how tailored probes and measurements improve over standard methods in the noisy regime. In our approach, we advocate the mindset of quantum-aided design, exploiting quantum technology to learn close to optimal, experimentally feasible quantum metrology protocols.

Keywords: variational quantum algorithms, quantum metrology, quantum technology, noisy intermediate scale quantum devices

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The first author was student at time of submission and would appreciate if the submission was taken into consideration for the student prize.

Extended Abstract

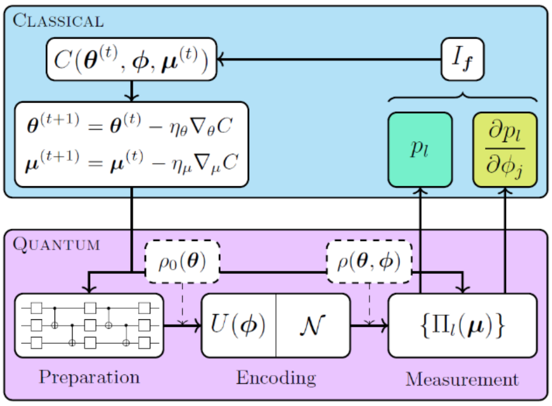

Figure 1 Illustration of the hybrid approach presented in this work to variationally optimize the probe state and measurement for a noisy unitary multi-parameter estimation problem.

Quantum metrology exploits non-classical effects to extend the precision of sensing methods beyond classical limits. To realize such advantages in noisy multi-parameter metrology scenarios, it is crucial to carefully design suitable quantum probes and measurements [1,2]. This is a highly challenging task and can quickly become classically intractable as it involves optimizations over exponentially large quantum states with possibly very intricate noise.

We propose to use the growing body of tools originating from the study of *noisy intermediate scale quantum (NISQ)* computers to overcome this obstacle. Our work provides a *variational quantum algorithm* for the optimization of quantum sensing protocols. It combines hybrid quantum-classical algorithms [3] with variational approaches that have already been used in the context of quantum metrology [4,5]. The algorithm is applicable to a wide range of realistic metrology problems in the general multi-parameter setting. It can be implemented *directly* on controllable quantum devices, including the targeted sensing platform, or on a near-term quantum computer, thereby circumventing the need to simulate large quantum systems classically.

A high-level description of our algorithm is sketched in Figure 1. In each step of the algorithm, a probe quantum state is generated by a parametrized quantum circuit. It undergoes a noisy unitary transformation, consisting of a unitary evolution encoding the physical parameters and an arbitrary, possibly non-local, quantum channel accounting for the system noise. The subsequent measurement is described by a parametrized positive-operator valued measure (POVM) resulting in measurement output probabilities depending on the parametrization of both the probe state and the POVM.

This concludes the part of the protocol that runs on the quantum device. This step is repeated a number of times to get accurate estimates of the output probabilities. These are then used to classically compute a cost function based on the Cramér-Rao bound that quantifies the estimation quality of the protocol. From this, both the state preparation circuit and the measurement procedure are updated to further increase the estimation quality based on gradient-descent techniques. The entire procedure is iterated until a minimum is reached, yielding a close to optimal sensing protocol within the variational manifold of probe state preparation and measurement procedure.

We note that our algorithm can account for classical postprocessing of the measured quantities via a possibly non-linear multivariate function. This is of key importance as the physical quantities (e.g. phases) are usually only acting as a proxy for the actual quantity of interest (e.g. the contraction of space that causes a phase difference in a gravitational wave interferometer).

We detail the necessary level of control that a sensing device needs to allow to be able to run the algorithm. If this level of control is not achievable, another quantum device or a general purpose NISQ computer can be used to run the algorithm by emulating the sensing protocol. We discuss how this emulation, especially of the involved error channels, can be performed through repeated sampling or use of ancillas. If the simulation of the noise channel is not possible, we provide a way to move some of the calculations to the classical side to alleviate hardware requirements. To allow the use of gradient-descent based techniques, we provide novel material on parameter-shift rules [6] for noise channels.

We then demonstrate the practical functioning of our approach through numerical simulations. We reproduce known results about the optimality of GHZ states for noiseless sensing of phase averages [7] and study the performance of local and shallow entangled probes for an experiment where three nitrogen vacancy centers are used to determine the position of a target spin.

We believe that our research has significant potential to impact the development of quantum technologies and to contribute to a shift in mindset that sees quantum technologies themselves as crucial tools to improve future generations of quantum technologies.

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