Entropic uncertainty relations and the quantum-to-classical transition

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As far as science is concerned the world is fundamentally quantum. Yet, this quantum nature is not commonly perceived in our everyday life, as many well described phenomena in microscopic systems do not seem to be present in the macroscopic world. Moreover, an essential difference between classical and quantum description of systems seems to further bisect the two classes: both use observables to describe measurements, but quantum ones must additionally obey Heisenberg's uncertainty principle (HUR). It implies that, given two observables A and B, a quantum state may not have their associated quantities simultaneously well defined. The question that naturally arises is how do we transition from a quantum to a classical paradigm. A naive answer is to simply increase the system's size, which does not suffice.

In this work, we shift our attention from HUR as a measure of "classicality" to entropic uncertainty relations (EUR). The EUR display the same physical interpretation as HUR while covering its shortcomings (such as state dependence or sensitivity to re-scaling). In this formulation, classicality can be achieved if the lower bound of the EUR approaches zero as the system increases in size, meaning a state could have both A and B simultaneously well defined.

For our analysis, we look particularly into normalized total magnetization observables in the directions x and z, that notably do not commute. Their usual description is simply the normalized sum of local magnetizations. The EUR analysis shows that we do not get the expected behavior on the lower bound of the inequality. Moreover, the bigger the system the worse this scenario gets, since the lower bound in the EUR depends linearly on the system's size N.

Our first measure is to restrict the class of states we are taking into consideration to spin-coherent states $|\Psi_N\rangle = |\Psi_1\rangle^{\otimes N}$, known to saturate the bound above. However, if we revisit the EUR restricting ourselves to spin-coherent states we still get a lower bound that increases with N, though at a lower rate.

The next adjustment is to adapt the model of macroscopic measurements. The usual description requires a complete knowledge of every qubit in the system, while we are only interested in the total magnetization along a certain axis. Moreover, it is unrealistic to expect these macroscopic measurements to be infinitely precise. Exploiting the natural degeneracy of these measurements, we combine these two notions into a macroscopic measurement description by bins. This means we are no longer measuring the probability of a state Ψ having a certain spin configuration, but rather of having a normalized total magnetization that falls inside a given interval (bin).

Considering states Ψ_N and modifications to total magnetization, we finally get the expected behavior: the sum of their associated entropies decreases with N. The right-hand side of the EUR approaches zero, indicating that the quantities associated to these two macroscopic measurements can be simultaneously well defined for a spin-coherent state. Moreover, given the Gaussian behavior of the associated probabilities, we would reach a similar result even if we increased the number of bins with the system size, but not faster than $\sqrt{N}$. A numerical analysis is also available in fig (1).

In this work, we show that the inclusion of imprecision on macroscopic measurements is required to observe a classical character in the system. Conversely, to preserve quantum features on large systems (e.g. large quantum computers) the number of outcomes in the measurements has to grow faster than $\sqrt{N}$.

Figure 1: **Sum of entropies for increasing system size.** As N increases, it becomes possible for coherent-spin states to have well-defined magnetization simultaneously in x and z directions.