Quantum causal correlations and non-Markovianity of quantum evolution

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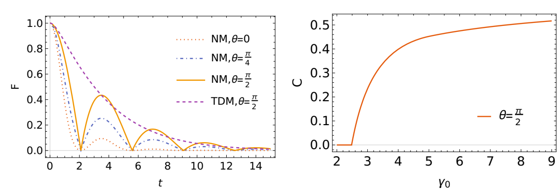
A novel non-Markovianity (NM) measure is introduced based on causality measure (CM)- recently introduced by Fitzimons et.al., Scientific reports, 5:18281, (2015) - which quantifies temporal (causal) correlations. The measure is justified with the example of a qubit interacting with a bosonic reservoir with Jaynes-Cummings type of interaction. Breakdown of monotonicity of CM is shown to be associated with negativity of the decay rate in the master equation hence to the non-Markovian nature of the channel.

In [1], NM measure was introduced based on temporal steerable weight which the authors prove to be monotonic under a Markovian completely positive trace preserving map. Recently, it was shown [2] that temporally non-separable correlations or quantum causal correlations [3], temporal steerable correlations, and Leggett-Garg inequality violating correlations form a hierarchy. In [4], I make use of monotonicity property of causality measure in the pseudo-density matrix formalism, to define a non-Markovianity measure and justify it with an example. *Correlations in time* can be formulated by associating different qubit Hilbert spaces $\mathcal{H}_{t_1}$ and $\mathcal{H}_{t_2}$ to the density matrix ρ_A at time t_1 and ρ_B at time t_2 respectively. Time evolution is generally obtained by using a completely positive trace preserving (CPTP) map $\mathcal{E}_{B\leftarrow A}$ such that $\rho_B = \mathcal{E}_{B\leftarrow A}[\rho_A]$. The state ρ_A may be thought of as an input to the quantum channel $\mathcal{E}$. A two-point PDM under any channel $\mathcal{E}$ can be given an alternative representation [3]: $\mathcal{P}_{\rm AB} = (I\otimes\mathcal{E})\left[\left\{\rho\otimes\frac{I}{2}, Q_{\sf swap}\right\}\right]$ where $Q_{\sf swap} := \frac{1}{2}\sum_{i=0}^{3}\sigma_i\otimes\sigma_i$ and $\{a,b\} = ab + ba$ and σ_i are Pauli-X,Y and Z operators with $\sigma_0 = I$. Above state shares all the features of a proper density matrix except that it may not be positive semi-definite. However, the PDM is Hermitian and has unit trace. Throughout the paper we consider one time use of channel acting on the initial qubit state ρ with measurements before and after the use of the channel. A causality measure is defined as $F = \log_2 \|\mathcal{P}_{\rm AB}\|_1$, based on trace norm of $\mathcal{P}$. A CPTP map $\mathcal{E}$ is CP-divisible in the sense that the condition $\mathcal{E}(t+\tau, t)\mathcal{E}(t,0) = \mathcal{E}(t+\tau,0)$ and associated with this is the equivalent condition $\mathcal{F}_{\mathcal{E}[\rho]}(t) \geq \mathcal{F}_{\mathcal{E}[\rho]}(t+\tau)$, for all τ is small increment in time. When these conditions are violated, the corresponding channel is said to non-Markovian. Based on this one can define a measure of non-Markovianity $\mathcal{M} := \max_{\rho}\int_{\sigma_{(\rho,\mathcal{E},t)}>0} dt\ \sigma_{(\rho,\mathcal{E},t)}$ where $\sigma_{(\rho,\mathcal{E},t)} = \frac{dF}{dt}$ for channel $\mathcal{E}$ acting on an initial state ρ, where the integration is done over positive slope of $\mathcal{F}$. As an example, breakdown of monotonicity of F is plotted in the above figure for the Jaynes-Cummings model in its time-dependent Markov (TDM) and non-Markov (NM) regimes, for various values of θ pertaining to the initial state $\rho = \begin{pmatrix} \sin^2(\theta) & \frac{1}{2}\sin(2\theta) \\ \frac{1}{2}\sin(2\theta) & \cos^2(\theta)\end{pmatrix}$.

One may observe the recurrences of F when $\mathcal{E}$ is NM. A normalized measure $C = \frac{\mathcal{M}}{1+\mathcal{M}}$ is plotted in Fig. 1(b) for the JC model against the coupling strength γ, for an initial state with $\theta = \frac{\pi}{2}$. When the measure $\mathcal{M} = 0$, the evolution is Markovian and when $\mathcal{M} > 0$, the evolution is NM.



Important Note: Despite the conclusions of the current version of [4], there are no disadvantages as such of PDM and hence of the non-Markovianity measure introduced. Therefore, PDM as well the measure are well defined for a system of *any* dimension. The manuscript [4] will be updated in near future.

[1] Shin-Liang Chen, Neill Lambert, Che-Ming Li, Adam Miranowicz, Yueh-Nan Chen, and Franco Nori. Quantifying non-markovianity with temporal steering. *Physical review letters*, 116(2):020503, 2016.
[2] Huan-Yu Ku, Shin-Liang Chen, Neill Lambert, Yueh-Nan Chen, and Franco Nori. Hierarchy in temporal quantum correlations. *Physical Review A*, 98(2):022104, 2018.
[3] Robert Pisarczyk, Zhikuan Zhao, Yingkai Ouyang, Vlatko Vedral, and Joseph F Fitzsimons. Causal limit on quantum communication. *Physical review letters*, 123(15):150502, 2019.
[4] Shrikant Utagi. Quantum causal correlations and non-markovianity of qubit evolution. *arXiv:2005.04129v1 [quant-ph]*, 2020.

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