Universal quantum computation using single qubit discrete-time quantum walk

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Abstract. Universal quantum computation can be realised using both continuous-time and discrete-time quantum walks. We present a version based on single qubit discrete-time quantum walk to realize multi-qubit computation tasks. The scalability of the scheme is demonstrated by using a set of walk operations on a closed lattice form to implement the universal set of quantum gates on multi-qubit system. We also present a set of experimentally realizable walk operations that can implement Grover's algorithm, quantum Fourier transformation and quantum phase estimation algorithms. Analysis of space and time complexity of the scheme highlights the advantages of quantum walk based model for quantum computation on systems where implementation of quantum walk evolution operations is inherent feature of the system.

Keywords: Quantum computation and Algorithm, Quantum walk

Quantum walk based quantum computing model was first introduced on unweighted graph using the continuous-time quantum walk [1] and a corresponding scheme using discrete-time quantum walk was later proposed [2]. Recently, we proposed a new scheme using a single qubit discrete-time quantum walk on a closed lattice setting [3].

In this work, we present a detailed extension of the simple, implementable quantum computing scheme using a single qubit discrete-time quantum walk which can be scaled to higher dimensions [3]. Along with the position Hilbert space on which the quantum walks are defined, the discrete-time quantum walk provides additional degree of freedom in the form of coin Hilbert space that can be exploited to achieve control over the states to perform computing operations. This model can be implemented on a photonic or lattice based quantum systems where one photon or free particle can act as coin that can be used to perform computation when entangled with the position space.

We propose the use of multiple sets of closed graph with four sites and four edges to act as a system with $2^{(N-1)}$-dimensional position space. Each graph is equivalent to two-qubit state and n-sets of closed graph provides 2n-qubit equivalent states. With the help of the coin and shift operations, the particle (coin) and the position state can be evolved into the desired output state.

We also demonstrate the effectiveness of our scheme by presenting a combination of quantum walk operations to implement the quantum algorithms like Grover's search algorithm, quantum Fourier transformation and phase estimation algorithms.

References

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