A mathematical framework for operational fine tunings

Lorenzo Catani and Matthew Leifer

Schmid College of Science and Technology, Chapman University, Orange, CA, 92866, USA

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All the features of quantum mechanics that emerge as inherently nonclassical in the framework of ontological models share a common aspect: they involve fine tuned properties – or, for simplicity, *fine tunings*. These are defined as properties that hold at the operational level, but do not hold at the ontological level. Let us consider a paradigmatic example. Bell's theorem can be explained by appealing to the failure of parameter independence, one of the components in the assumption of Bell's local causality [1]. Accepting this explanation would mean that no signaling holds at the level of the operational statistics, but it does not hold at the ontological level, due to the presence of superluminal causation. In this case no signaling would emerge as a fine tuned property, because it would only arise from a fine tuning of the ontic parameters. Accepting fine tuned properties is problematic in a scientific theory, because they characterize nature with a conspiratorial connotation, meaning that there are some features that are not available for us to use and experience, even if they are present in nature. This denies the core idea of science and its empiricist roots.

In 2015 Wood and Spekkens [2] introduced the notion of fine tunings in the framework of classical causal models [3] and showed that there are no explanations of Bell's theorem free of fine tunings. The requirement of no causal fine tuning reads as a criterion for the most natural causal explanation of the observed conditional independencies between the variables. In contrast, in this work we refer to fine tunings that are purely operational, without any assumption on the underlying causal structure. In brief, the condition of no operational fine tuning requires the operational equivalences between the statistics of experiments to be preserved at the ontological level. Here the ontological level is defined by the notion of ontic extension [4], which removes the causal assumptions from the standard ontological model framework [5]. Common examples of operational fine tunings are the ones that arise from the violation of parameter independence and generalized noncontextuality.

In summary, we provide a rigorous mathematical framework, developed also in the language of category theory and functors, that characterizes operational fine tunings. In addition to accounting for all the known operational fine tunings, the framework describes more general ones, thus setting the ground for formulations of further no-go theorems. In light of our framework and the distinction we draw between operational and causal fine tunings, we analyze the notion of Bell's local causality, that is composed by an operational fine tuning – parameter independence – and a purely causal fine tuning – outcome independence [6]. In this way we deepen the understanding of the relation between nonlocality and generalized contextuality, where the former is not just an example of the latter because it can be obtained, unlike contextuality, by involving a purely causal fine tuning.

The current work originates a proper research program, where the next step consists of formulating a resource theory for operational fine tunings. This would also allow us to witness and quantify the presence of fine tunings in information processing tasks and quantum computational protocols. Finally, the very foundational motivation for studying and characterizing fine tunings is to ultimately develop a new ontological framework for quantum theory free of fine tunings.

References

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