Extended Abstract: Compositional resource theories of coherence, arXiv:1911.04513

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The understanding and manipulation of coherence within quantum systems is central to the development of quantum technologies. More than this though, deeply understanding coherence is central to our understanding of quantum theory itself. Indeed, whilst loss of coherence in a quantum system is a critical problem in the development of quantum computers, it is essential for the emergent classical world in which we live our day to day lives [26, 49, 50]. Developing better mathematical tools for addressing quantum coherence is therefore pertinent to both the foundations of physics as well as quantum engineering.

One of the key developments on the abstract formalism of resource theories has been the work of [18]. This utilises the mathematics of category theory and process theories [19, 40] to study resource theories in quantum theory and beyond. A key benefit of this approach is that it is intrinsically and explicitly compositional in nature. In particular, the mathematics of category theory comes with a natural diagrammatic representation which makes reasoning intuitive and greatly simplifies many complex calculations. For instance, this diagrammatic reasoning has been used to provide a natural environment for describing measurement-based quantum computation [15], quantum teleportation [14], and quantum secret sharing [16] among many others [19].

The study of resource theories of coherence has, to date, largely taken place from a more convex-geometric perspective, utilising tools such as majorization [38, 39] and Schur-convex functions [23, 38]. For example see the works of [1, 9, 13, 37, 47, 48]. However, this approach is somewhat limited in scope, and makes it difficult to generalise beyond resource theories of coherence for single system quantum states. Nowhere is understanding the manner in which multipartite, channel, and measurement coherence manifests as a resource more important than in a large-scale quantum computer, where multipartite systems evolve through the application of channels to eventually be measured. Hence understanding how coherence in such settings can be controlled and quantified—that is, used as a resource—is vital for achieving the promise of large-scale quantum computing.

In this work we take a complementary perspective and work entirely process-theoretically, equivalently we work entirely categorically, that is to say, diagrammatically. This has many advantages: it unifies many existing approaches, for example, providing a unified treatment of speakable and unspeakable coherence; it provides a general treatment of resources theories of coherence in a multipartite setting, highlighting the deficiencies within existing definitions when going to this regime; it is immediately applicable to the study of coherence of measurements, channels, general quantum instruments, and higher-order processes; and, it is immediately generalisable to the situation in which there are multiple notions of coherence, for instance, relative to different bases. Moreover, it naturally extends to the study of coherence within alternative physical theories [21, 22], such as generalised probabilistic theories [7, 10, 24, 25] and epistemically restricted theories [2, 17, 44, 45]. This last point is useful as it provides the ability to characterise coherence in operational and physical terms, rather than via specific features of the mathematical formalism of quantum theory—such as Hilbert spaces and complex numbers.

More importantly, however, than the application to specific alternative theories, is that this general approach demonstrates that the basic essence of coherence can be captured in terms of composition of processes, and hence, that the information-theoretic approach should not be taken to be primitive.

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