Landauer's principle at zero temperature
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Landauer's bound relates changes in the entropy of a system with the inevitable dissipation of heat to the environment. The bound, however, becomes trivial in the limit of zero temperature. In this work we derive a stricter bound which remains non-trivial even at low temperatures. We consider the usual setup with a thermal environment at temperature T and the only extra information required to use this new bound is the dependancy of the environment heat capacity with the temperature (nothing is said about the state of the system or the kind of system-environment interaction).

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In a setup where a system interacts with an uncorrelated thermal environment at temperature T, Landauer's principle (LP) [1] connects changes in the system entropy ΔS_S with the heat dissipated to the environment ΔQ_E. More precisely, it states that $\Delta Q_E \geq -T\Delta S_S$. The connection between entropy, information and computation gives some interesting interpretations to this principle, like providing guidelines on the ultimate dissipative costs of computation and establishing the minimum heat cost necessary for purifying the state of a system.

In recent years, several generalizations of LP have been put forth [2–5]. However, in the limit $T \to 0$, LP, as well as its many generalizations become trivial; stating only that $\Delta Q_E \geq 0$, irrespective of ΔS_S. To see why this is unsatisfactory consider, for example, a qubit coupled to a radiation field. If the qubit starts in a mixed state and the field starts in a vacuum, then larger changes to the qubit entropy are accompanied by larger flows of heat to the environment (by spontaneous emission), indicating that there should be a dependancy of ΔQ_E with ΔS_S.

In this work we derive the following generalization of Landauer's principle

$$\Delta Q_E \geq Q(\mathcal{S}^{-1}(-\Delta S_S)). \tag{1}$$

where $Q(\tau) = U_E^{th}(\tau) - U_E^{th}(T)$ and $\mathcal{S}(\tau) = S_E^{th}(\tau) - S_E^{th}(T)$ while U_E^{th} and S_E^{th} are the equilibrium internal energy and equilibrium entropy of the environment as a function of its temperature.

We prove that this generalization has the desirable properties of always being stricter than the usual LP bound and of remaining non-trivial in the $T \to 0$ limit. Moreover, the equilibrium properties of the environment used can all be boiled down to the heat capacity.

In its form (1), our new bound is somewhat abstract. But it can be made explicit for specific environments. A particularly illuminating example, is that of emission onto a one-dimensional waveguide, that yields

$$\Delta Q_E \geq -T\Delta S_S + \frac{3\hbar c}{\pi L k_B^2}\Delta S_S^2, \tag{2}$$

which holds for an arbitrary system coupled in an arbitrary way to the waveguide. Here c, k_B and L are the speed of light, Boltzmann's constant and the length of the waveguide respectively. It's immediatly clear that this bound is always stricter than LP and remains informative even when $T \to 0$. In particular, it shows that it is impossible to change the system von Neuman entropy (ΔS_S) without an ensuing heat exchange ΔQ_E and that the correction is more relevant, the smaller the waveguide is.

Other examples included in the work are emission to a phonon environment, emission to a gapped environment and a comparison between our new bound and the usual LP bound on simulations of the Rabi model. Some perspectives on how feasible it is to experimentally measure this effect are also considered in the case of emission to a waveguide.

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