Genuine multipartite nonlocality is intrinsic to quantum networks

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Quantum entanglement and nonlocality are inextricably linked. While necessary, entanglement is not always sufficient for nonlocality in Bell scenarios. We derive sufficient conditions for entanglement to produce genuine multipartite nonlocality (GMNL) in networks. We find that any connected network of bipartite pure entangled states is GMNL, independently of topology and amount of entanglement shared. We deduce that all pure genuine multipartite entangled (GME) states are GMNL in the sense that measurements exist on finitely many copies of any GME state that yield a GMNL behaviour. Our results pave the way towards feasible manners of generating GMNL using any connected network.

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Correlations between quantum particles may be much stronger than those between classical particles, and their applications are manifold [1]. A necessary, but in general not sufficient, condition to achieve nonlocality is quantum entanglement [2, 3]. Still, pure entangled states of any number of parties can always create bipartite nonlocality [4, 5]. In the multipartite regime, the structure of correlations is much richer. Studying the relationship between genuine multipartite entanglement and nonlocality is relevant for quantum networks and many-body physical systems, and this is the focus of our work.

We show that the nonlocality arising from networks of bipartite entangled states is a generic property and manifests in its strongest form, genuine multipartite nonlocality (GMNL). Specifically, we obtain that any network where the parties are connected by bipartite pure entangled states gives rise to GMNL. As a consequence, we show that all pure genuine multipartite entangled (GME) states are GMNL in the sense that measurements can be found on finitely many copies of any GME state to yield a GMNL behaviour. In fact, we prove that for an n-partite GME state, $(n-1)$ copies are always sufficient for n-way GMNL.

While GMNL states may be difficult to create experimentally, our work reveals that bipartite entanglement distributed in a connected network is sufficient to obtain a GMNL distribution. It was already known that a star network of maximally entangled states is GMNL [6], but we generalise this result and make it independent of the amount of entanglement shared as well as of the network topology. Thus, we show GMNL is an intrinsic property of networks of pure bipartite entangled states.

Further, there are known mixed GME states that are not GMNL [7, 8]—some are even fully local [9]. However, recent results show that, for pure n-qubit symmetric states [10] and all pure 3-qubit states [11], GME implies GMNL (at the single-copy level). By showing that finitely many copies of any pure GME state are GMNL, our work thus tightens the relationship between entanglement and nonlocality in the multipartite regime.

Our construction exploits the fact that the set of non-GME states is not closed under tensor products, i.e. GME can be superactivated by taking tensor products of states that are not entangled across different bipartitions. Thus, GME can be achieved by distributing bipartite entangled states among dierent pairs of parties. To obtain our results, we extend the superactivation property [12–14] from the level of states to that of probability distributions, i.e. GMNL can be superactivated by taking Cartesian products of probability distributions that are local across different bipartitions. In fact, when considering copies of quantum states, we only consider local measurements performed on each copy separately, thus pointing at a stronger notion of superactivation to achieve GMNL.

Recent work has introduced stronger concepts than GME such as genuine network entanglement [15] which, roughly speaking, rules out states that are a tensor product of non-GME states. Our results can also be interpreted in this context: one could hope that states that are GME but not genuine network entangled might be detected in a device-independent manner by not passing GMNL tests; however, this turns out not to be the case. Any distribution of pure bipartite states, even with arbitrarily weak entanglement, always displays GMNL as long as all parties are connected.

Thus, by exploiting the multipartite nonlocality properties of bipartite entangled states, our work paves the way towards a feasible way of generating GMNL in networks based only on topological considerations, and at the same time shows GMNL can be extracted from any pure GME state.

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