Self-testing Bell inequalities from the stabiliser formalism and their applications

F. Baccari[1], R. Augusiak[2], I. Šupić[3], J. Tura[1] and A. Acín[4,5]

[1] Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, 85748 Garching, Germany
[2] Center for Theoretical Physics, Polish Academy of Sciences, Aleja Lotników 32/46, 02-668 Warsaw, Poland
[3] Département de Physique Appliquée, Université de Genève, 1211 Genève, Switzerland
[4] ICFO - Institut de Ciencies Fotoniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels (Barcelona), Spain
[5] ICREA - Institucio Catalana de Recerca i Estudis Avançats, Pg. Lluis Companys 23, 08010 Barcelona, Spain

Abstract

I will introduce a powerful tool to construct self-testing Bell inequalities from the stabiliser formalism and present two applications in the framework of device-independent certification protocols. Firstly, I will show how the method allows to derive Bell inequalities maximally violated by the family of multi-qubit graph states and suited for their robust self-testing. Secondly, I will present how the same method permits to introduce the first examples of subspace self-testing, a form of certification that the measured quantum state belongs to a given quantum error correction code subspace, which remarkably includes also mixed states.

Keywords: Self-testing, device-independent protocols, entanglement, graph states, error correction codes

Bell inequalities constitute a key tool in quantum information theory: they not only allow one to reveal nonlocality in composite quantum systems, but, more importantly, they can be used to certify relevant properties thereof in a device-independent manner [1].
The highest level of device-independent certification is referred to as self-testing: it can be seen as a way of certifying both the state produced and the measurements performed by some given quantum devices, by simply looking at the resulting correlations [2].
Bell-inequalities-based self-testing methods rely on the properties of the state and measurements that can be inferred by the fact that some observed correlations violate maximally a Bell inequality. Such approach to self-testing is very convenient for experimental scenarios, since to date it is the only one that can be related to certification statements that are robust against realistic noise levels [3]. However, the task of finding a Bell inequality maximally violated by a specific quantum state is very challenging, and no general techniques have been introduced so far.
Here I will present a method to derive self-testing Bell inequalities tailored to states defined by a stabiliser formalism. In particular, I will show how such a method can be successfully applied in two scenarios: the family of multiqubit graph states and some subspaces related to error correction codes.
In the first case, I will show a general construction of Bell inequalities that are maximally violated by any graph state and can be used for its robust self-testing. Apart from their theoretical relevance,
these inequalities offer two main advantages from an experimental viewpoint: (i) they present a significant reduction of the experimental effort needed to violate them, as the number of correlations they contain scales only linearly with the number of observers; (ii) numerical results indicate that the self-testing statements for graph states derived from our inequalities tolerate noise levels that are met by present experimental data.
In the second case, I will present the first self-tests of entangled subspaces - those spanned by the quantum codewords of the five-qubit and the topic error correction codes. I will show that all quantum states maximally violating a suitably chosen Bell inequality must belong to the corresponding code subspace, which remarkably includes also mixed states.
Such a result implies that self-testing can be associated to the certification of more complex entanglement structures than single states. On a more fundamental level, the introduced Bell inequalities identify face structures in the set of quantum correlations of nonzero dimension, showing at the same time that self-testing methods are not limited to extremal points in the quantum set, but can also be attributed to higher-dimensional flat objects in its boundary such as faces. In particular, here I will show that in the simplest multipartite Bell scenario, the quantum set has a face of dimension three for any number of parties. Lastly, I will introduce a framework to address the problem of robust self-testing in the case of entangled subspaces and present some numerical results for the case of the five-qubit code.
For both the considered scenarios, the resulting Bell inequalities are qualitatively different from those obtained from previous approaches[4-7]. In particular, the presented method is the first that allows to make a direct connection to self-testing statements.
The presentation will be based on the results in [8] and [9].

References

[1] N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, and S. Wehner, Reviews of Modern Physics 86, 419
[2] D. Mayers and A. C. Yao, Quantum Information & Computation 4, 273
[3] J. Kaniewski, Physical review letters 117(7), 070402
[4] O. Gühne, G. Tóth, P. Hyllus, and H. J. Briegel, Physical Review Letters 95, 120405
[5] O. Gühne and A. Cabello, Physical Review A 77, 032108
[6] A. H. Shenoy and R. Srikanth, Journal of Physics A: Mathematical and Theoretical, 52(9):095302
[7] M. Waegell and J. Dressel, npj Quantum Information, 5(1):1–8
[8] F. Baccari, R. Augusiak, I. Šupić, J. Tura, J and A. Acín, Physical Review Letters, 124(2), 020402
[9] F. Baccari, R. Augusiak, I. Šupić, J and A. Acín, arxiv:2003.02285