There and back again: A circuit extraction tale

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Translations between the quantum circuit model and the measurement-based one-way model are useful for verification and optimisation of quantum computations. By expressing both one-way computations and circuits in the common language of the ZX-calculus, we give the first algorithm that can translate any one-way computation with *extended gflow*, i.e. allowing measurements in the XY, XZ, and YZ-planes of the Bloch sphere, into an ancilla-free circuit. Circuits can be optimised by translating them into a one-way computation, optimising this, and translating back.

Keywords: Measurement-based quantum computing, ZX-calculus, quantum circuit optimisation

Computations in the measurement-based one-way model [12] are represented as measurement patterns [5, 6]. These specify the resource state for the computation (a graph state), as well as the measurements which drive the computation, and the corrections which compensate for the non-deterministic outcomes of quantum measurements. A measurement pattern is deterministically implementable if and only if it has a property called *generalised flow*, or *gflow* [2]. Much research has focused on measurement patterns in which all measurements lie in the XY-plane of the Bloch sphere (spanned by the eigenstates of Pauli-X and Y), as this case is computationally universal [12]. Yet the one-way model also allows measurements in the XZ- and YZ-planes [2]. A gflow on a pattern with measurements in more than one plane is often called an *extended gflow* [9]. Previous algorithms translating measurement patterns into circuits either require ancillas [1, 13, 14] or apply only to patterns where all measurements are in the XY-plane [7, 11].

Following [7, 8], we use the ZX-calculus [3, 4] to represent both measurement patterns and quantum circuits. Translations between the two models of quantum computation can then be performed by graphically rewriting ZX-diagrams. ZX-calculus diagrams which closely correspond to measurement patterns are called *MBQC-like* (e.g. Figure 1a), those corresponding to circuits are called *circuit-like* (e.g. Figure 1b). An MBQC-like ZX-diagram is said to have (extended) gflow if and only if the associated measurement pattern has (extended) gflow.

In this work, we show that certain graphical rewrites preserve the property of a ZX-diagram being MBQC-like, as well as the property of it having extended gflow. Thus, we may use graphical rewriting to find different measurement patterns representing the same computation; for example, we can remove Clifford measurements as in [10]. We give the first algorithm that can take any measurement pattern which has extended gflow and transform it into a circuit. The algorithm builds up on a previous circuit-extraction algorithm for MBQC-like ZX-diagrams [7] that works only if all measurements are in the XY-plane. It is also the most general currently-known algorithm for extracting a quantum circuit from unitary ZX-diagrams. Quantum circuits can be optimised with respect to properties such as T-count by translating them into measurement patterns, rewriting the pattern, and then re-extracting a circuit using our algorithm.

This is an extended abstract, the full paper can be found at `https://arxiv.org/abs/2003.01664`.

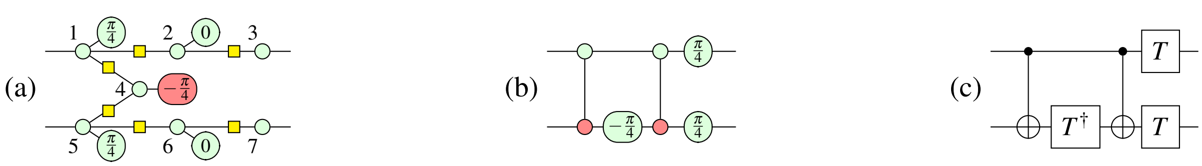

Figure 1: The MBQC-like ZX-diagram with extended gflow (a) is rewritten by our algorithm (plus some mild simplification) to the circuit-like ZX-diagram (b). The latter corresponds to the quantum circuit (c).

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References

[1] Anne Broadbent & Elham Kashefi (2009): *Parallelizing quantum circuits*. Theoretical Computer Science 410(26), pp. 2489–2510, doi:10.1016/j.tcs.2008.12.046.

[2] Daniel E Browne, Elham Kashefi, Mehdi Mhalla & Simon Perdrix (2007): *Generalized flow and determinism in measurement-based quantum computation*. New Journal of Physics 9(8), p. 250, doi:10.1088/1367-2630/9/8/250.

[3] B. Coecke & R. Duncan (2008): *Interacting quantum observables*. In: Proceedings of the 37th International Colloquium on Automata, Languages and Programming (ICALP), Lecture Notes in Computer Science, doi:10.1007/978-3-540-70583-3_25.

[4] B. Coecke & R. Duncan (2011): *Interacting quantum observables: categorical algebra and diagrammatics*. New Journal of Physics 13, p. 043016, doi:10.1088/1367-2630/13/4/043016. ArXiv:quant-ph/09064725.

[5] Vincent Danos, Elham Kashefi & Prakash Panangaden (2007): *The Measurement Calculus*. J. ACM 54(2), doi:10.1145/1219092.1219096.

[6] Vincent Danos, Elham Kashefi, Prakash Panangaden & Simon Perdrix (2009): *Extended Measurement Calculus*, pp. 235–310. Cambridge University Press, doi:10.1017/CBO9781139193313.008.

[7] Ross Duncan, Alex Kissinger, Simon Perdrix & John van de Wetering (2019): *Graph-theoretic Simplification of Quantum Circuits with the ZX-calculus*. arXiv:1902.03178.

[8] Ross Duncan & Simon Perdrix (2010): *Rewriting measurement-based quantum computations with generalised flow*. In: International Colloquium on Automata, Languages, and Programming, Springer, pp. 285–296, doi:10.1007/978-3-642-14162-1_24.

[9] Nidhal Hamrit & Simon Perdrix (2015): *Reversibility in Extended Measurement-Based Quantum Computation*. In Jean Krivine & Jean-Bernard Stefani, editors: Reversible Computation, Springer International Publishing, pp. 129–138, doi:10.1007/978-3-319-20860-2_8.

[10] M. Hein, J. Eisert & H. J. Briegel (2004): *Multiparty entanglement in graph states*. Physical Review A 69(6), p. 062311, doi:10.1103/PhysRevA.69.062311.

[11] Jisho Miyazaki, Michal Hajdušek & Mio Murao (2015): *Analysis of the trade-off between spatial and temporal resources for measurement-based quantum computation*. Physical Review A 91(5), p. 052302, doi:10.1103/physreva.91.052302.

[12] Robert Raussendorf & Hans J. Briegel (2001): *A One-Way Quantum Computer*. Phys. Rev. Lett. 86, pp. 5188–5191, doi:10.1103/PhysRevLett.86.5188.

[13] Raphael Dias da Silva & Ernesto F Galvão (2013): *Compact quantum circuits from one-way quantum computation*. Physical Review A 88(1), p. 012319, doi:10.1103/physreva.88.012319.

[14] Raphael Dias da Silva, Einar Pius & Elham Kashefi (2013): *Global quantum circuit optimization*. arXiv:1301.0351.