Critical Quantum Metrology
with a Finite-Component Quantum Phase Transition

Louis Garbe[1], Matteo Bina[2], Matteo Paris[2], and Simone Felicetti[3]

[1] *Université de Paris, Laboratoire Matériaux et Phénomènes Quantiques, UMR 7162 CNRS, Paris, France.*
[2] *Quantum Technology Lab, Dipartimento di Fisica Aldo Pontremoli,*
Università degli Studi di Milano, I-20133 Milano, Italy.
[3] *Departamento de Física Teórica de la Materia Condensada, and Condensed Matter Physics Center (IFIMAC),*
Universidad Autonoma de Madrid, Spain.

Short abstract: Quantum critical systems in proximity of phase transitions exhibit a divergent susceptibility, suggesting that an arbitrarily high precision may be achieved when they are used as probes to estimate a physical parameter. However, such an improvement in sensitivity is counterbalanced by the critical slowing down, which implies an inevitable growth of the protocol duration. Here, we design metrological protocols based on a novel kind of phase transitions observable in finite-component quantum optical systems. We show that, in spite of the critical slowing down, critical quantum optical probes can achieve a quantum-enhanced time scaling of the sensitivity in frequency-estimation protocols.

Keywords: Quantum Metrology, Quantum Optics, Phase Transitions, Quantum Technologies.

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Metrological quantum advantage is usually defined as the precision that can be achieved in the estimation of a physical parameter, as a function of the number of probe systems. The maximum precision achievable with classical systems scales linearly with the number of probes, while the optimal quantum strategy achieves a quadratic scaling, known as the Heisenberg limit.
It is well known that some quantum systems can undergo a critical phase transition [1] in the thermodynamic limit, where the number of constituents becomes macroscopic. In proximity of a quantum phase transition the system susceptibility diverges, leading to an apparent super-Heisenberg scaling of the estimation precision [2,3]. However, the divergence of the susceptibility is counterbalanced by the vanishing energy gap, which makes the estimation protocol slower and slower as the critical point is approached. It has only recently been shown that, once the estimation time is put back in the analysis, the optimal Heisenberg scaling is recovered [4].

In this contribution **[5]**, we present metrological protocols that make use of a finite-component quantum phase transition, which has been predicted to take place also for a system composed of a single two-level atom coupled to a single bosonic mode. In the finite-component case, the thermodynamic limit is replaced by a diverging scaling imposed on the Hamiltonian parameters. Under realistic noise conditions, we compare the metrological performances of standard Ramsey interferometry protocols with two strategies that exploit the finite-size criticality: an adiabatic following and a driven dissipative setting.
In particular, we evaluate analytically the quantum Fisher information, which is a measure of the optimal estimation precision achievable with a given estimation protocol. We focus on the scaling of the quantum Fisher information with respect to the estimation time, both for short- and long-time evolution limits. We find that this approach allows one to measure both bosonic and spin frequency with a favourable time scaling, in spite of the critical slowing down.
Finally, we discuss how the proposed protocols could be applied to implement a quantum magnetometer using nowadays atomic and solid-state quantum technologies, such as trapped ions [6] or superconducting quantum circuits [7].

Our results show that critical quantum-optical systems represent a compelling tool for quantum metrology. In addition, we unveil the metrological potential of finite-component quantum phase transitions, a result that have both practical and fundamental consequences. Finally, by focusing on the time scaling and on a finite-component system, our analysis challenges the standard framework in which the fundamental resources needed to achieve quantum advantage are usually assessed.

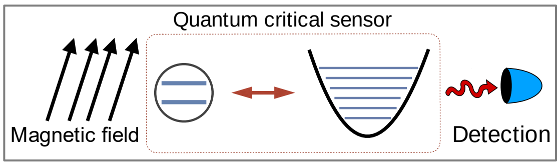

Figure Schematic representation of a quantum critical magnetometer composed of a two-level system coupled to a quantum harmonic resonator.

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