Solving Vehicle Routing Problem Using Quantum Approximate Optimization Algorithm

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Abstract. Here, we describe the usage of Quantum Approximate Optimization Algorithm (QAOA), a quantum-classical heuristic, to solve the combinatorial optimization task called Vehicle Routing Problem (VRP). We outline its Ising formulation and solve it by minimizing its simulated Ising Hamiltonian using the IBM Qiskit platform. Here, we attempt to find solutions for VRP problem instance: (n, k), with n locations and k vehicles. We find that the performance of QAOA is dependent upon the classical optimizer used, the way parameters are initialized, the number of steps p in which an adiabatic path is realized, and on the problem instance itself.

Keywords: Ising Model, Combinatorial Optimization, Quantum Approximate Algorithms, Variational Quantum Algorithms

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In general, quantum computing devices are supposed to have a computational advantage over classical processors by using quantum resources such as superposition and entanglement. However, the computational capabilities of these current generation quantum processors also known as Noisy Intermediate-Scale Quantum (NISQ) devices, are considerably restricted due to their intermediate size (in terms of qubits count), limited connectivity, imperfect qubit-control, short coherence time and minimal error correction. Hence, they are only able to run algorithms with limited circuit depth. These belong to the class of quantum-classical hybrid variational algorithms and are proven effective at solving combinatorial optimization problems. Here, in this paper, we use Quantum Approximate Optimization Algorithm to solve the Vehicle Routing Problem (VRP).

1. VRP

Vehicle Routing Problem is an NP-hard combinatorial optimization problem. Any problem instance (n, k) of VRP involves k vehicles, and n−1 locations (other than the depot D). Its solution is the set of routes in which all of the k vehicles begin and end in the D, such that each location is visited exactly once. The optimal route is the one in which the total distance travelled by k vehicles is least.

2. ISING FORMULATION

To solve a problem instance (n, k) of VRP using QAOA, we first need to map it to the minimization, i.e., finding the ground state of an Ising Hamiltonian Hc. We write the formulation for VRP as the following and then an energy functional HVRP:

$$VRP(n,k) = min_{\{x_{ij}\}_{i\to j}\in\{0,1\}} \sum_{i\to j} w_{ij}x_{ij}$$

$$H = A\sum_{i=0}^{n-1}[z_{S[i]}z_{S[i]}^T + z_{T[i]}z_{T[i]}^T]\mathbf{x}^2 + w^T\mathbf{x} - 2A\sum_{i=1}^{n-1}[z_{S[i]}^T + z_{T[i]}^T]\mathbf{x} - 2Ak[z_{S[0]}^T + z_{T[0]}^T]\mathbf{x} + 2A(n-1) + 2Ak^2$$

Here, we are skipping details about proof and constraints for brevity. This H can now be mapped the spin Ising formulation. QAOA can be thought of as a coarsely trotterized adiabatic time evolution in p steps to $|\psi_{GS}^{HC}\rangle$ i.e., the ground state of a Hamiltonian Hc which encodes the problem from $|\psi_{GS}^{HM}\rangle$ i.e., the ground state of the Hamiltonian Hm which is known and easier to prepare. In gate-model quantum computation this means that starting

from some initial product state $|\psi_{GS}^{HM}\rangle$, we apply a parameterized gate sequence to produce the state $|\psi_{GS}^{HC}\rangle$. This is done using a set of parameters {β, γ}, which are provided by a classical processor, and also optimized by an optimization routine based on the result of energy measurement for final state $|\psi^{HC}\rangle$.

3. SIMULATION AND RESULTS

We have executed QAOA using IBM Qiskit's both noise-free and noisy backends to solve VRP for three problem instances: (4, 2), (5, 2), and (5, 3), where each (n, k) represents a problem with n location and k vehicles with a distance matrix D representing the squared euclidean distances between locations. One needs N=n×(n−1) qubits to encode the problem instance, i.e., state of each qubit represents the possibility of an edge between two nodes.

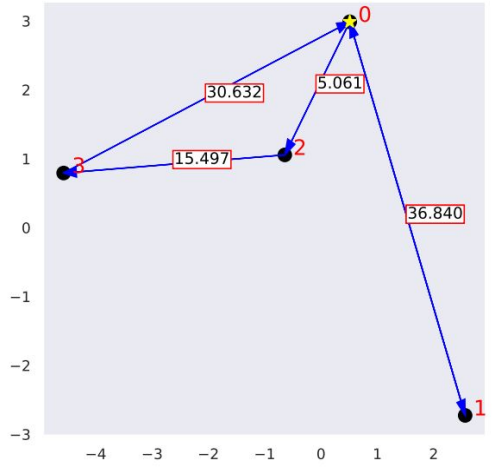

Figure 1 Visualization of the solution state indexed 779 = "110100001100" for D = [[0. 36.84 5.06 30.63], [36.84 0. 24.55 63.22], [5.06 24.55 0 15.50], [30.63 63.22 15.50 0]]. The cost is: C = 30:632 + 15:497 + 5:061 + 2×36:840 = 124:871. Here, the node with yellow star denotes the depot, or the origin.

Through the results of our simulations, we conclude that in general, for a finite value of p, there is no guarantee that the solution achieved by QAOA corresponds to the most optimal solution of the original combinatorial optimization problem. This is because we are trying to guess the adiabatic time evolution path using p steps. So, the first straightforward reason could be that the chosen value p does not produce a good enough guess. Then, another reason which could explain failure of QAOA at larger values of p could be the emergence of new local minimums in our solution energy-landscape which traps both gradient-free and gradient-based optimizers and make them converge prematurely. Moreover, it was seen that on noisy simulators, the noise-based errors affect both the fidelity of state: $|\beta, \gamma\rangle$, prepared by a quantum routine, and the minimized expectation value of $\hat{H}_{cost}$, i.e., $\langle \hat{H}_{cost} \rangle$.

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