Hierarchy of Theories with Indefinite Causal Structure

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Abstract We present a diagrammatic representation of the Causaloid framework and study the Third level (Meta) compression in detail. We show that there is a hierarchy of theories with respect to Meta compression and characterise its general form. Next, we populate the hierarchy. The standard theory of circuits forms the simplest case, which we can express diagrammatically through Duotensors, following which we construct Triotensors for the next rung in the hierarchy.

Keywords: Indefinite Causality, Post-Quantum, Diagrammatic Calculus, Quantum Foundations

The problem of Quantum Gravity is to find a theory that reduces to General Relativity on the one hand and Quantum Theory on the other. General Relativity is a theory in which the causal structure is dynamical. In Quantum Theory, any quantity that is dynamical is subject to quantum indefiniteness (for example, the path of a particle is indefinite as to which slit it goes through). This means we expect *indefinite causal structure* in a theory of Quantum Gravity.

A reasonable starting point in finding a theory of Quantum Gravity is to find a mathematical framework that is general enough to accommodate theories with indefinite causal structure. Any theory having a state that evolves in time cannot accommodate indefinite causal structure since it presupposes that we have a foliation into spacelike hypersurfaces. The causaloid formalism, presented in [1-3], might be regarded as a general probability theory for theories with indefinite causal structure. Since then, the Quantum Combs Framework [5] and the Process Matrix Framework [6] were formulated to study indefinite causality.

In the causaloid framework a notion of regions is established, this being based on all the data that might be recorded during an experiment. The framework has three levels of compression. By *compression* we mean that, given probabilities for some limited set of circumstances, we can calculate probabilities for a bigger set of circumstances. The first level (or Tomographic) compression pertains to a single region and reproduces the generalised probability theories landscape. The second level (or Compositional) compression relates to the composition of two or more regions. The third level (or Meta) compression concerns equations relating the matrices that further encode compositional compression. To facilitate exposition, we set up a diagrammatic approach to the causaloid framework.

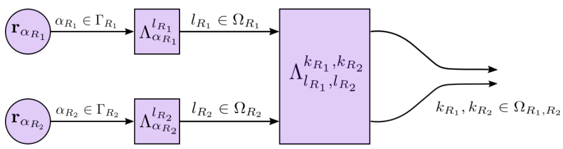

Figure: Tomographic and Compositional Compression in the Causaloid Framework

In this work [7-8] we study Meta compression and find that there is a hierarchy of theories with respect to Meta compression. For example, two-region composition is sufficient for the standard theory of circuits (which includes classical and quantum cases) which pertains to the simplest case in this hierarchy, and is governed by a general form:

$$\Omega_{R_1,R_2,\ldots,R_n} = O\left(\sigma\left(\mathbb{I}^n \prod_i^n P_i^{\times\left(\binom{n}{2}-n\right)}\right) \prod_i^n \Omega_{R_i}^{\times\binom{n}{2}}\right)$$

We illustrate this by explicitly showing how to put the circuits into this framework using the *duotensor* [4] approach to the theory of circuits. Following this, we will construct the *triotensors* pertaining to the next case in the hierarchy concerning compositional compression of three regions. This concerns circuits with hyper-edges that connect three boxes (rather than two boxes in as in standard circuits). Finally we discuss the implications of this work going forward for the field of indefinite causal structures.

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