Universal modifications to time dilation in quantum clocks

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Abstract

The general theory of relativity associates a proper time with objects via their world-lines - classical trajectories forbidden by quantum theory. Here we demonstrate that in the post-Newtonian limit, "good" quantum clocks with classical states of motion, experience time dilation as dictated by general relativity. For nonclassical states of motion, however, we find that quantum interference leads to a significant modification to general relativistic time dilation. Moreover, we show that the ignorance of the clock's state of motion leads to a larger uncertainty in time measurements, a consequence of the entanglement of the clock's time and motional degrees of freedom.

Key words: Quantum clocks, quantum gravity, time dilation

A successful theory of quantum gravity is one of the most important unsolved problems in theoretical physics. Arguably, many difficulties in this pursuit arise due to a lack of understanding of the underlying nature of time; in particular the conflicting roles of time in general relativity and quantum physics. General relativity treats time as a relative property of observers, while quantum physics treats it as a universal parameter. Furthermore, general relativity assigns a proper time to an object according to its world line; however, such well defined trajectories are forbidden in quantum mechanics. The status of time in non-relativistic quantum physics is also a subject of perpetual debate. Ordinary quantum physics postulates time as parameter, unlike other objects like position and momentum which are treated as observables or operators in Hilbert space. The present work addresses these ambiguities; particularly the question, how can a proper time be assigned to delocalised objects described by quantum theory?

We take an operational approach to this question using quantum clocks [1]. While there are several proposed models of quantum clocks, no physical observable can perfectly track the parameter t in quantum mechanics (i.e. work as an idealised clock), as was noted by Pauli [2]. However, it is indeed possible to have a finite dimensional clock that exhibits *quasi-ideal* behaviour [3]. For the purpose of our work, starting with minimal assumptions, we introduce a novel characterisation for the accuracy of *arbitrary* quantum clocks. This characterisation takes the form of an *error* operator - defined as the deviation from the canonical commutation relation between the Hamiltonian of the clock and the time operator. We consider slowly moving quantum clocks in a weakly-curved spacetime, or the post-Newtonian approximation of general relativity. In other words, we study the behaviour of arbitrary clocks when they follow the laws of both quantum physics and general relativity. We show that all "good" clocks under our characterisation (i.e. those for which the error operator is small in norm) show classical time dilation when their state of motion is classical (Gaussian). However, introducing quantum interference by taking a spatial superposition of states of motion, we find a significant discrepancy between the average time measured by the clock, and that predicted by general relativity - a consequence of incorporating both general relativity and quantum mechanics in our model. This is also true for an idealised quantum clock (i.e. for which the error operator is zero). We elaborate our findings with several physical examples including the Salecker-Wigner-Peres (SWP) clock [4, 5] and the quasi-ideal clock [3]. For the SWP clock, surprisingly we find that for "good" clock states (i.e. eigenstates of its "time operator"), there are no relativistic effects on time measurement at the lowest order, even when the state of motion is classical.

Going further, we show that relativistic coupling of temporal and kinematic degrees of freedom of the clock leads to a significant increase in the uncertainty with which it measures time accurately. We demonstrate that this increase is due to the temporal state losing information due to its entanglement with the kinematic degrees of freedom of the clock. We show that this uncertainty can be decreased by gaining information about the kinematic degrees of freedom, by performing coarse grained measurements on them. We further show that in the limit of increasing information gain, one recovers the non-relativistic uncertainty in time; as if there were no coupling between temporal and kinematic degrees of freedom due to relativity. Our analysis shows that relativity mandates knowing how a clock moves, or one must pay a penalty in its precision.

References

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