

Contextual advantage for noisy one-shot classical communication assisted by entanglement*

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We study the noise-robust generalisation of an earlier proposed strategy [1] that considers the enhancement of one-shot zero-error capacity of certain Kochen-Specker hypergraph-based classical channels assisted by noiseless entanglement. Our general analysis considers the enhancement of the one-shot success probability of sending a fixed set of classical messages over general classical channels (e.g., [6]) assisted by noisy entangled states and/or local measurements. We demonstrate the necessity and sufficiency of contextuality for quantum advantage, identifying contextuality as the key nonclassical feature for this task. We further highlight graph-theoretic properties of certain classical channels and bound the enhancement with graph-theoretic witnesses of contextuality [4, 5, 3].

Keywords: Contextuality, Nonclassicality, Classical Communication, Classical Channels, Pseudo-Telepathy Games, One-shot communication, Kochen-Specker Theorem, Graph-theoretic frameworks, Graph-theoretic witness, Bipartite entanglement, Entanglement Assistance, Zero-error, Noisy measurements, Noisy entangled states, Noise-robust contextuality, Quantum advantage, Noise attenuation, Success probability of communication, Hypergraph invariants, Weighted max-predictability

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The central promise of advances in quantum technologies is a quantum advantage over what's possible classically. But identifying the source of this improvement is not trivial. While several nonclassical resources within quantum theory are well-studied, one would ideally like features that can be tested operationally (i.e., directly from data) under minimal assumptions about the structure of the theory. These features would not only characterize and distinguish quantum theory from classical theories but would also provide insights about the source of quantum advantage in as theory-independent a manner as possible. In this contribution, we demonstrate that contextuality (i.e., violation of the assumption of noncontextuality) does exactly this in the context of entanglement-assisted one-shot classical communication.

The framework for noncontextuality that we use, developed by Spekkens [7], posits that operational indistinguishability implies ontological indistinguishability. This criterion for ontological models constrains the statistics that can be simulated by them and quantum theory yields statistics that violates this restriction. We show that in entanglement-assisted classical communication, this violation yields a quantum advantage.

While the original Kochen-Specker theorem that introduced contextuality was restricted to ideal projective measurements and used a hypergraph-uncolourability argument, recent work by Kunjwal and Spekkens [4, 3] operationalizes contextuality and provides noise-robust contextuality witnesses using a hypergraph invariant. They show that in the noncontextual regime, the average correlation between preparation and measurement pairs (with some equivalence-structure hypergraphs) is upper bounded by a hypergraph invariant called the *weighted max-predictability*, denoted β : $\text{Corr}_{NC} \leq \beta < 1$. A violation of this inequality indicates contextuality of quantum theory. On the other hand, for a classical channel, Shannon [2] showed that the maximum number of zero-error messages is the maximal independence number of the classical channel's hypergraph¹, denoted by α . Noticing that a graph-theoretic invariant (α) bounds the zero-error capacity, *and* that in quantum theory such bounds can be exceeded, one is motivated to ask if one-shot communication can be enhanced with the use of quantum resources. Cubitt *et al.* [1] explore this connection and provide a communication protocol assisted by Kochen-Specker sets that improves the channel's one-shot zero-error capacity. While this is a fundamental proof-of-concept, enhancing the zero-error capacity in this manner is not noise robust and thus experimentally infeasible. We therefore generalize this protocol to allow for noisy shared entangled states and/or local measurements. Instead of the one-shot zero-error capacity, we use the success probability of communication for a fixed number of messages as our figure of merit. With this, our notion of quantum advantage gets formalized as exceeding the maximal classical success probability. In our work, contextuality emerges as a nonclassical feature that drives this advantage.

We first provide a general framework for entanglement-assisted one-shot classical communication. We demonstrate that preparation noncontextuality is a necessary and sufficient restriction for recovering the classical shared randomness regime. We then restrict Bob's inference strategy by assuming he is oblivious to the exact channel probabilities, but retains knowledge of the channel's hypergraph structure. We call this Bob's *context-independent guessing* (CIG). This motivates operational equivalences between Bob's measurement procedures, thus paving the way for a connection with contextuality. We demonstrate necessity and sufficiency of contextuality for quantum advantage and show that $\eta^{min} + \text{Corr}(1 - \eta^{min}) \leq S \leq \eta^{max} + \text{Corr}(1 - \eta^{max})$ ². Corr here witnesses contextuality on Bob's side by considering the correlation between the states steered by Alice onto Bob's side and Bob's local measurements. When we have projective measurements and maximally entangled states, $\text{Corr} = 1$, and thus $S = 1$ irrespective of the channel probabilities, recovering the zero-error result of Cubitt *et al.* In the special case where $\eta^{min} = \eta^{max} \equiv \eta$, we have that $S = \eta + \text{Corr}(1 - \eta)$ and $\text{Corr} > \beta$ implies that S exceeds the classical success probability.

¹In a channel hypergraph, the input symbols are represented by vertices and hyperedges are the confusability sets associated with the output symbols

²where η^{min} and η^{max} are the minimum and maximum confusability of two input symbols from Bob's perspective and depend *only* on the classical channel

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