Semi-device-dependent blind quantum tomography

Ingo Roth, Jadwiga Wilkens, Dominik Hangleiter, Jens Eisert

To achieve the promising advantages of quantum technologies one must engineer individual quantum components with an enormous precision. From an engineering perspective, improving such *noisy intermediate scale quantum (NISQ) devices* [1, 2] requires advanced and flexible diagnostic techniques to extract actionable advice on how to improve a device. One of the most basic diagnostic tasks is the extraction of tomographic information about quantum states from experimentally measured data. Indeed, at the heart of every quantum computation is the preparation of a quantum state. Quantum state tomography can therefore provide valuable information for improving quantum devices beyond a mere benchmarking of their correct functioning [3].

However, in any such endeavour one encounters the following fundamental challenge: In order to arrive at an accurate state estimate, most tomography schemes rely on measurement devices that are calibrated to a very high precision. At the same time, a precise and detailed characterization of a measurement device requires accurate state preparations. But characterizing the state preparation was the goal to begin with. We are trapped in a vicious cycle. This constitutes a fundamental obstacle to the improvement of quantum devices.

In this work, we break the vicious cycle. We observe that in reasonably controlled quantum devices, commonly encountered quantum states exhibit a natural structure: they are close to being pure. We leverage this natural property to prove that one can simultaneously learn an unknown calibration of a measurement device and a low-rank quantum state. We thus arrive at what we coin a *semi-device-dependent* scheme in which the dependence on the measurement apparatus is significantly softened.

In order to achieve this goal, we formulate the *blind tomography problem* as the recovery task of a highly structured signal. In mathematical terms, the blind tomography task that we solve is to infer a vector ξ of n calibration parameters and a rank-r quantum state ρ from data of the form

$$y = \mathcal{A}(\xi \otimes \rho), \tag{1}$$

where $\mathcal{A}$ is a linear map describing the measurement model. Such data arise, for example, when we perform measurements of a multi-qubit Pauli operator W_k with expectation value $\mathrm{Tr}\,[\rho W_k] =: [A_0(\rho)]_k$, that are disturbed by coherent errors which affect only some of the Pauli matrices, a setting that is natural in ion trap platforms. For instance, if the natural measurement basis is the computational Z-basis and Hadamard gates incur a small coherent error with angle ϕ around the Z-axis, then a measurement of a Pauli-Y observable will be replaced by an additive mixture of Pauli-Y and Pauli-X measurement weighted with certain calibration coefficients ξ_i. In many situations the daunting uncertainty about the device calibration is small and can be approximated as a linear deviation from an empirically known calibration baseline, resulting in a faulty measurement

$$y = \xi_0 \mathcal{A}_0(\rho) + \sum_i \xi_i \mathcal{A}_i(\rho) =: \mathcal{A}(\xi \otimes \rho). \tag{2}$$

Our formulation of the blind tomography problem allows us to exploit and further develop a powerful formal machinery from signal processing to devise a scalable self-calibrating state tomography scheme that comes with rigorous performance guarantees. More specifically, we further develop so-called *iterative hard-thresholding (IHT)* algorithms [4] – a key work horse for solving structured linear inverse problems in in the field of model-based *compressed sensing* [5, 6]. We begin by showing a negative result, namely, that any such approach, when applied to the blind tomography problem (1) directly, does not yield an efficient algorithm for the blind tomography problem.

We proceed by observing that the blind tomography problem can naturally be relaxed to a related recovery problem, which we call the *sparse demixing tomography problem*. For this problem we develop an algorithm, the SDT-Algorithm. For the toy setting in which the measurement data is given by expectation values of highly unstructured (Gaussian) observables we not only show that it requires a close-to-optimal number of measurements, but also that it quickly converges to the optimal solution. This shows that the SDT algorithm efficiently solves the blind tomography problem in an idealized scenario. Viewed from a signal-processing perspective, these rigorous results generalize work on the demixing and recovery of multiple low-rank matrices of Ref. [7] to sparse mixtures.

Complementing these conceptual and rigorous insights, we numerically demonstrate that blind quantum tomography is possible and practically feasible using constrained alternating optimization by exploiting low-rank assumptions in the practical setting described above.

References

[1] J. Preskill, *Quantum computing in the NISQ era and beyond*, Quantum **2**, 79 (2018).

[2] A. Acín, I. Bloch, H. Buhrman, T. Calarco, C. Eichler, J. Eisert, D. Esteve, N. Gisin, S. J. Glaser, F. Jelezko, S. Kuhr, M. Lewenstein, M. F. Riedel, P. O. Schmidt, R. Thew, A. Wallraff, I. Walmsley, and F. K. Wilhelm, *The quantum technologies roadmap: a European community view*, New J. Phys. **20**, 080201 (2018).

[3] J. Eisert, D. Hangleiter, N. Walk, I. Roth, D. Markham, R. Parekh, U. Chabaud, and E. Kashefi, *Quantum certification and benchmarking*, ArXiv e-prints (2019), 1910.06343 [quant-ph].

[4] T. Blumensath and M. E. Davies, *Iterative thresholding for sparse approximations*, J. Four. An. App. **14**, 629 (2008).

[5] R. G. Baraniuk, V. Cevher, M. F. Duarte, and C. Hegde, *Model-based compressive sensing*, IEEE Trans. Inf. Th. **56**, 1982 (2010).

[6] S. Foucart and H. Rauhut, *A mathematical introduction to compressive sensing* (Springer, 2013).

[7] T. Strohmer and K. Wei, *Painless Breakups–Efficient Demixing of Low Rank Matrices*, Journal of Fourier Analysis and Applications , (2017), 10.1007/s00041-017-9564-4.