Phase-space inequalities: theory and experiment

Martin Bohmann,[1, *] Nicola Biagi,[2, 3] Jan Sperling,[4] Alessandro Zavatta,[2, 3] Marco Bellini,[2, 3] and Elizabeth Agudelo[1]

[1] *Institute for Quantum Optics and Quantum Information - IQOQI Vienna, Austrian Academy of Sciences, Boltzmanngasse 3, 1090 Vienna, Austria*

[2] *Istituto Nazionale di Ottica (CNR-INO), L.go E. Fermi 6, 50125 Florence, Italy*

[3] *LENS and Department of Physics&Astronomy, University of Firenze, 50019 Sesto Fiorentino, Florence, Italy*

[4] *Integrated Quantum Optics Group, Applied Physics, University of Paderborn, 33098 Paderborn, Germany*

We introduce the concept of phase-space-inequality conditions for the verification of nonclassicality. This approach allows us to reveal quantum correlations even if the corresponding phase-space distributions are nonnegative. A fundamental relation between these inequality conditions and correlations measurements is given. The strength of the method is demonstrated by certifying quantum correlations from experimental data where other methods fail to do so.

The identification and characterization of nonclassical state of light is a central task in quantum optics and photonic quantum information. Nonclassicality as a resource is of major importance for quantum technologies such as quantum metrology, communication, or entanglement generation. Therefore, it is crucial to develop efficient and experimentally accessible tools for the characterization of nonclassical light. One possibility of identifying genuine nonclassical features is using the framework of quasiprobability distributions [1]. Alternatively, inequality conditions based on moments of observables can be used [2]. Both approaches come with their own advantages and drawbacks.

Here, we introduce a framework which unifies the certification of quantum correlations through quasiprobability distributions and inequality conditions. In this way, we demonstrate a deep connection between correlation measurements and phase-space distributions and device nonclassicality conditions which exploit the advantages of both approaches. Firstly, we derive conditions based on Chebyshev's integral inequality which relate different phase-space distributions to each other [3]. Importantly, this approach allows us to certify nonclassicality even if the involved phase-space distributions are nonnegative. Additionally, we show that the derived phase-space inequalities are closely related to correlation measurements which are widely used for certifying quantum correlations in quantum optics.

Secondly, we unify the the notions of quasiprobabilities and matrices of correlation functions [4]. The method developed here correlates arbitrary phase-space functions at arbitrary points in phase space, including multimode scenarios and higher-order correlations. Thus, it provides a profound conceptual insight and unites these two fundamental tools of testing for nonclassicality. To demonstrate the versatility of our technique, the quantum characteristics of discrete- and continuous-variable, single- and multimode, as well as pure and mixed states are certified.

We illustrate the strength and practicality of the presented methods by applying them to experimental data [5]. In particular, we use different phase-space inequality conditions to certify the nonclassical character of lossy and noise single-photon states. The single-photon state is generated via heralding detection from a spontaneous parametric down-conversion source and is recorded via balanced homodyne detection. Different loss and noise levels are introduced. Remarkably, we can detect nonclassicality in parameter regions where other established methods fails to do so.

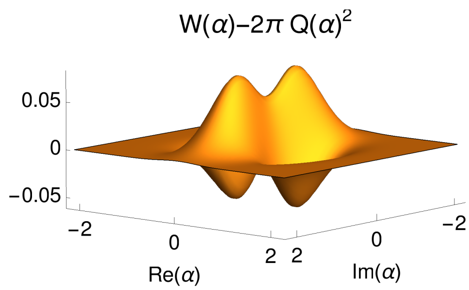

FIG. 1. Phase-space inequality of a squeezed state. Negativities certify nonclassicality despite the fact that the underlying distributions (Q and W) are nonnegative.

* martin.bohmann@oeaw.ac.at

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