Contextuality-by-default for behaviours in compatibility scenarios

Alisson Tezzin[1], Rafael Wagner[1], and Bárbara Amaral[1]

[1] *University of São Paulo, Institute of Physics, São Paulo, SP, Brazil*

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Extended abstract

The compatibility-hypergraph approach to contextuality [1, 2, 3] and the contextuality-by-default approach [4, 5] are usually seen as products of entirely different views on how physical measurements and measurement contexts should be understood: the latter is based on the idea that a physical measurement has to be seen as a collection of random variables, one for each context containing that measurement, while the imposition of the non-disturbance condition as a physical requirement in the former precludes such interpretation of measurements. The aim of our work is to present both approaches as entirely compatible ones and to introduce in the compatibility-hypergraph approach important ideas which arises from contextuality-by-default. In order to obtain such compatibility we show how a behaviour in a scenario (scenarios and behaviours are the main components of the compatibility-hypergraph approach) associates to each measurement a contextual collection of random variables, i.e., we show how a behaviour in a scenario defines a system, which is the main component of the contextuality-by-default approach. After establishing it we introduce in the compatibility-hypergraph approach the non-degeneracy condition, which is the analogous of consistent connectedness [5]: a behaviour is non-degenerate when the system defined by it is consistent connected. These behaviours are exactly those which associate just one random variable to each measurement (more precisely, a collection of random variables with the same distribution), which means that non-degeneracy draws the line between the two views on measurements described above. The set of non-degenerate behaviours defines a polytope, therefore it can be characterized by means of linear inequalities, just as we do with non-disturbance and standard non-contextuality [3]. We prove that any non-disturbing behaviour is non-degenerate and which the reverse is false, implying that non-degeneracy is a condition usually weaker then non-disturbance. The second concept we introduce is "contextuality in the extended sense", i.e., contextuality-by-default in itself. We call non-contextual in the extended sense any behaviour whose system has a maximally non-contextual description [5]. We prove that a behaviour is non-contextual in the standard sense if and only if it is non-degenerate and non-contextual in the extended sense. It means that the standard definition of contextuality is the extended one when restricted to non-degenerate behaviours. Finally, we use extended scenarios and behaviours, introduced in [6], to shed new light on our results.

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