Efficient description of many-body systems with Matrix Product Density Operators

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Abstract. Matrix Product States form a powerful ansatz for the simulation of one-dimensional quantum systems that are in a pure state. Their power stems from the fact that they faithfully approximate states with a low amount of entanglement, the "area law". However, in order to accurately capture the physics of realistic systems, one generally needs to apply a mixed state description. In this work, we establish the mixed state analogue of this characterization: We show that one-dimensional mixed states with a low amount of entanglement, quantified by the entanglement of purification, can be efficiently approximated by Matrix Product Density Operators.

Keywords: entanglement, tensor networks, MPDO

Multipartite quantums state are difficult to describe theoretically because of a so-called *curse of dimension*. This refers to the fact that the state space of such systems grows exponentially with the system size. Therefore generic quantum systems with more than a few tens of particles cannot be exactly described on classical computers, due to memory restrictions. To address this problem, there has been an extensive search for methods that allow to *approximate* multipartite states with good accuracy while requiring fewer computational resources. One such method is tensor networks, which describe multipartite states using only a number of parameters polynomial in the number of parties.

A natural question arises about which states can be approximated using tensor networks. This question was partially answered by Verstraete and Cirac [1]. They found a sufficient condition for a one-dimensional pure multipartite state to be efficiently approximable by Matrix Product States (a type of one-dimensional tensor network). The condition said that the entanglement of the state across every bipartition (as measured by the α-Rényi entropy of the reduced state) may grow at most logarithmically with the system size.

Since the publication of the aforementioned result, there was little progress in generalizing it. In particular, the result of Verstraete and Cirac applies only to pure states. However, "impure" mixed states appear much more commonly in nature as well as in the lab. In the present work [2], we set out to extend the result to mixed states.

Our work can be divided into several steps. First, we identify a quantity to use in the criterion instead of the α-Rényi entropy of the reduced state (which loses the meaning of entanglement for mixed states). We use the α-Rényi entanglement of purification, which refers to the purifications of the mixed state to quantify the amount of both classical and quantum correlations present. We show that small entanglement of purification across a bipartition implies that there exists a low-rank approximation across that bipartition. If such approximation exists for each bipartition, we can employ a scheme to "merge" those into one approximation to the original state with low operator-Schmidt rank (also known as bond dimension in the tensor network language) across every bipartition.

In the end we obtain a theorem similar to what was derived in [1], but for mixed states.

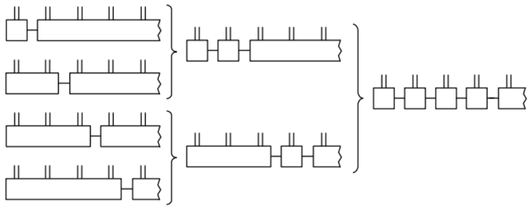

Figure 1 A tree-like scheme used to construct a tensor network from low-rank approximations.

References

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