Bounding the quantum capacity with flagged extensions

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arXiv preprint arXiv:2008.0246 (link: http://arxiv.org/abs/2008.0246)

Abstract. In this article we consider flagged extensions of channels that can be written as convex combination of other channels, and find general sufficient conditions for the degradability of the flagged extension. An immediate application is a bound on the quantum and private capacities of any channel being a mixture of a unitary operator and another channel, with the probability associated to the unitary operator being larger than 1/2. We then specialize our sufficient conditions to flagged Pauli channels, obtaining a family of upper bounds on quantum and private capacities of Pauli channels. In particular, we establish new state-of-the-art upper bounds on the quantum and private capacities of the depolarizing channel, BB84 channel and generalized amplitude damping channel. Moreover, the flagged construction can be naturally applied to tensor powers of channels with less restricting degradability conditions, suggesting that better upper bounds could be found by considering a larger number of channel uses.

Keywords: quantum shannon theory, quantum capacity, flagged extensions, pauli channels

Protecting quantum states against noise is a fundamental requirement for harnessing the power of quantum computers and technologies. In a transmission line or in a memory, noise is modeled as a quantum channel, and several accesses to the channel together with careful state preparation and decoding can protect quantum information against noise. The quantum capacity of a channel is the maximal amount of qubits which can be transmitted reliably, per use of the channel. The quantum capacity [1,2,3] of a channel can then be obtained as a limit for large n of the coherent information per use of the channel, for n uses of the channel. However, the potential super-additivity of the coherent information hinders the direct evaluation of the quantum capacity. The existence of an algorithmically feasible evaluation of the quantum capacity remains as one of the most important open problems in quantum Shannon theory, while finding computable upper or lower bounds on the quantum capacity constitutes important progress.

Extending previous results, in this work we formulate sufficient conditions to obtain non-trivial upper bounds on the quantum capacity, by degradable flagged extensions. A flagged extension of a channel that can be written as convex combinations of other channels is such that the receiver gets, together with the output of one of the channels in the convex combination, a flag carrying the information about which of the channels acted. While flagged extensions with orthogonal flags were already considered in [4,5], following [6,7] we also consider non-orthogonal flags. By specializing the new degradability conditions we obtain state-of-the-art upper bounds on the quantum capacity for two important Pauli channels. the depolarizing channel and BB84 channel, and for the generalized amplitude damping channel, improving the results of [6,7]. The bounds we obtain are not necessarily the best bounds available with these techniques, being good guesses among all the instances of flagged channels that satisfy the sufficient conditions. In fact, we obtain an infinite sequence of optimization problems depending on the number of uses of the channel, each of which gives a bound on the capacity. It is not clear if a phenomenon analogue to superadditivity appears in this scenario. Even with one use of the channel, different choice of Kraus operators give different bounds. This work paves the way to a systematic study of bounds on the quantum capacity with flagged extensions, which proved to be a powerful tool for this problem.

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