Persistence of Topological Phases in Non-Hermitian Quantum Walks

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Discrete-time quantum walks (DTQWs) are known to exhibit exotic topological states and phases. Physical realization of quantum walks in a noisy environment may destroy these phases. We investigate the behavior of topological states in quantum walks in the presence of a lossy environment. The environmental effects in the quantum walk dynamics are addressed using the non-Hermitian Hamiltonian approach. We show that the topological phases of the quantum walks are robust against moderate losses. The topological order in one-dimensional (1D) split-step quantum walk persists as long as the Hamiltonian is $\mathcal{PT}$-symmetric. Although the topological nature persists in two-dimensional (2D) quantum walks as well, the $\mathcal{PT}$-symmetry has no role to play there. Furthermore, we observe the noise-induced topological phase transition in two-dimensional quantum walks.

Keywords: Quantum Walks, Topological Matter, Non-Hermitian System, $\mathcal{PT}$- symmetry.

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I. 1D DTQW

Quantum walks are the quantum analog of classical random walks [1] where a quantum walker propagates on a lattice and the direction of propagation is conditioned over the state of the spin. The simplest DTQW consists of a quantum walker over a 1D lattice whose evolution is governed by a conditional shift operator T and a coin flip operator $R(\theta)$ for a real parameter θ. A more enriched class of 1D DTQW is split-step quantum walk (SSQW), which involves splitting the conditional shift operator T into left-shift ($T_\downarrow$) and right-shift ($T_\uparrow$) operators, separated by an additional coin toss $R(\theta_2)$ [2]. The resulting time evolution operator for 1D SSQW reads

$$U_{\text{ss}}(\theta_1, \theta_2) = T_\downarrow R(\theta_2) T_\uparrow R(\theta_1) \tag{1}$$

Motivated by the experimental results [3], we extend 1D SSQW to non-hermitian domain by introducing scaling operator $G = e^{\gamma\sigma_z}$ [4] which accounts for the loss and gain in the system. This results in the following time evolution

$$U_{\text{ss}}^{\text{NU}}(\theta_1, \theta_2, \gamma) = T_\downarrow G_\gamma R(\theta_2) T_\uparrow G_\gamma^{-1} R(\theta_1). \tag{2}$$

Since, $U_{\text{ss}}^{\text{NU}}(\theta_1, \theta_2, \gamma)$ is translational invariant, we go to momentum (quasi) basis by performing fourier transform and write corresponding generator, $H_{\text{NU}}(\theta_1, \theta_2, \gamma)$ such that $U_{\text{ss}}^{\text{NU}}(\theta_1, \theta_2, \gamma) = e^{-iH_{\text{NU}}(\theta_1,\theta_2,\gamma)}$ [2] which reads

$$H_{\text{NU}}(\theta_1, \theta_2, \gamma) = \bigoplus_k E(k)\, \hat{\mathbf{n}}(k) \cdot \sigma, \tag{3}$$

with quasi-energy $E(k)$. Note, that for $\gamma \neq 0$, G_γ as well as U becomes non-unitary and the corresponding generator non-hermitian. 1D SSQW is known to exhibit topological phases which are charaterized using winding number, $W = 0, 1$ [2] which are shown in Fig. 1(a). Due to the quantum nature of these topological phases, any loss in the system might lead to the destruction of the topological order.

II. 2D DTQW

We further extend our analysis to a 2D DTQW. There exists many variant of 2D DTQW. For our purpose, we define a 2D DTQW on a triangular lattice which consists of three conditional shift operators separated by coin-flip operations. We can define an equivalent 2D DTQW on a square lattice and using cyclic property with a time evolution operator given as

$$U_{2D}(\theta_1, \theta_2) = T_y R(\theta_1) T_y R(\theta_2) T_x R(\theta_1) T_x. \tag{4}$$

In 2D, we introduce loss and gain in one of the direction (say x) which results in non-hermitian dynamincs and the resulting time evolution operator reads

$$U_{2D}^{\text{NU}}(\theta_1, \theta_2, \gamma) = T_y R(\theta_1) T_y R(\theta_2) G_\gamma T_x R(\theta_1) G_\gamma^{-1} T_x.$$

Here also, we can go to quasi-momentum basis to define an effective Hamiltonian and the study topological properties. The topological phases has been studied in 2D DTQW and characterized by a topological invariant known as Chern number C [2]. It has been shown that 2D DTQW supports topological phases with $C = \pm 1, 0$ Fig. 2(a).

III. RESULTS

We study [5] the behavior of the topological phases in 1D SSQW and 2D DTQW by introducing a nonzero scaling factor γ which, essentially, makes the system non-Hermitian. In 1D SSQW, we find that the topological phases are unaffected even when the system is non-Hermitian (i.e., $\gamma \neq 0$), as far as the system possesses a real spectrum following the $\mathcal{PT}$-symmetry [6]. However, the topological nature of the system vanishes asymptotically as we cross the exceptional point γ_c, which can be seen in Fig. 1(b), 1(c). In 2D DTQW, we observe the persistence of the Chern number C as well until the scaling factor γ reaches a critical value. However, unlike the 1D case, we cannot associate any symmetry breaking with the point where the topological phase transition happens due to the absence of the symmetry in 2D DTQW. Unlike 1D SSQW, in this case we observe sharp transition. One more interesting point to note is that if we take particular values of θ_1 and θ_2 and increase scaling, it further shows noise-induced topological phase transition which was absent in 1D. We plotted C with scaling factor γ in Fig. 2(b), 2(c).

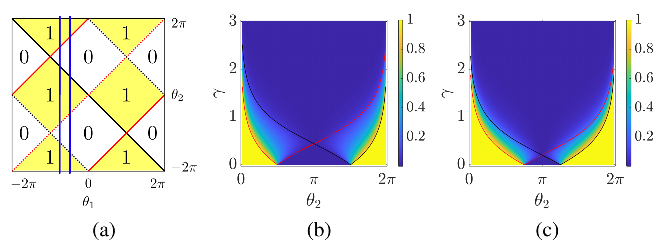

FIG. 1. (Color online) (a) Different topological phases realized in 1D SSQW as a function of θ_1 and θ_2. The two vertical blue lines correspond to two values of θ_1 which we considered. Winding number for lower energy band W_- as a function of γ and θ_2, and (b) $\theta_1 = -\pi/2$ (c) $\theta_1 = -3\pi/4$. The system size is taken to be $N = 100$. The red and black lines in all of the panels represent γ_c for $(k, E) = (0, 0)$ and $(k, E) = (\pi, 0)$, respectively.

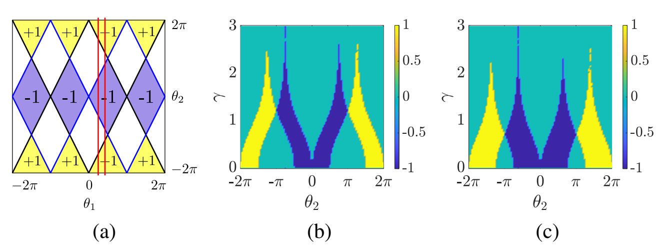

FIG. 2. (Color online) (a) Topological phases which exist in 2D DTQW for different values of θ_1 and θ_2. Effect of γ on Chern number is plotted with varying θ_2 for (b) $\theta_1 = \pi/4$ (c) $\theta_1 = 3\pi/8$. The system size is taken to be 50.

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