Continuous monitoring of energy in quantum open systems

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Abstract.

We propose a method to continually monitor the energy of a quantum system. We show that by having some previous knowledge of the system's dynamics, but not all of it, one can use the measured energy to determine many other quantities, such as the work performed on the system, the heat exchanged between the system and a thermal reservoir, the time dependence of the Hamiltonian of the system as well as the total entropy produced by its dynamics. We have also analyzed how this method is dependent on the quality factor of the measurements employed.

Keywords: quantum thermodynamics; open quantum system; quantum trajectories; stochastic thermodynamics

The field of thermodynamics is one of the most interesting and peculiar of all physics. It has given us the knowledge to build thermal engines and thus to reach the level of development that is seen in today's world. Its study has allowed us to understand the notion of irreversible processes and the arrow of time [1]. While thermodynamical properties of macroscopic systems are widely understood, the focus on microscopic systems is relatively new.

On the realm of stochastic thermodynamics, one iconic paper by Alicki has defined the form of the laws of thermodynamics for microscopic systems [2]. In this regime, the concept of thermodynamics is extended to a single stochastic trajectory of a microscopic system in phase space. In each realization of the same thermodynamical process, the system may undergo different trajectories, and quantities such as work, heat, and entropy production are no longer perfectly defined: they become trajectory dependent and, therefore, also stochastic [3–5]. In such cases, the second law of thermodynamics is associated with the central fluctuation theorem [6], $\langle e^{-\Delta S_{tot}} \rangle = 1$, where $\langle . \rangle$ is to be understood as the average over many trajectories. Also note that, just as the thermodynamical law itself, the central fluctuation theorem also has many formulations. An interesting example is the Jarzynski equality $\langle e^{-\mathrm{W}/k_B T} \rangle = \langle e^{-\Delta \mathrm{F}/k_B T} \rangle$ [7].

The goal of this work [8] is to propose a method to continually monitor the energy of a quantum system and to relate the monitored energy to its thermodynamics. In particular, we show that, by having some previous knowledge of the system's dynamics, one can use the measured energy to determine quantities such as the work performed on the system, the heat exchanged between the system and a thermal reservoir, the time dependence of the system's full Hamiltonian, and the total entropy produced by its dynamics.

The system analyzed is a two-level system subjected to a time-varying σ_z Hamiltonian and coupled to a thermal bath. The protocol to continually monitor the energy is based on the scheme of Roncaglia–Cerisola–Paz [9]. In this scheme a generalized quantum measurement is performed at a single time. We analyze the effects of imperfect measurements and define a quality factor for them. We show that a good quality factor allows to determine completely the Hamiltonian. Moreover, a careful analysis is presented about the relation of the quality factor and the central fluctuation theorem. We show that the measured value for the entropy production obeys an equation similar to the Jarzynski equality with a correction term that tends to zero as the quality factor increases.

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