Koopman Classical Mechanics as a guide to Quantum Mechanics

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Abstract: Koopman's Hilbert space formalism for Classical Mechanics includes a noncommutative algebra of measurements that can be interpreted in a natural classical way and that is, as a Hilbert space formalism, as capable as Quantum Mechanics of modeling any collection of measurements, experiments, and analysis of results. Koopman CM and QM can be thought of as different approaches to the same Hilbert space mathematics of states and noncommutative measurements, which can help to minimize the weirdness of QM significantly.

Keywords: Koopman classical mechanics; interpretation of quantum mechanics.

Extended abstract: The algebra of measurements in Koopman's Hilbert space formalism for CM includes a noncommutative algebra of derivations that can be constructed using the Poisson bracket, which generate transformations of the phase space, as well as the familiar commutative algebra of multiplicative operators[1]. We can discuss multiple measurements, experimental contexts, and transformations of experimental raw data using Koopman CM Hilbert spaces and operators as well as we can using QM Hilbert spaces and operators [with the principal difference being that the spectrum of the Liouvillian operator that generates timelike evolution in CM is unbounded, whereas the spectrum of the Hamiltonian operator that generates timelike evolution in QM is bounded below.]

The confirmation that a classical probabilistic state is well-supported by the statistics of experimental raw data can be subtle, but there has never been the same uncertainty as there is in QM about what happens when a particular measurement result is recorded: when a coin that is thrown once is recorded as a head or tail, we typically do not immediately update the statistical state. Unless only a very few trials are possible, decision theory and parameter estimation strategies most often will wait until the statistics of many throws give us a relatively better reason to make a change.

Furthermore, when we model actual joint measurements in Koopman CM, we always use commutative operators, because the relative statistics will certainly be in the range [0..1], so the joint probabilities generated by a formalism must satisfy the same constraint. This constraint adopted for QM gives a detailed mathematical expression to Bohr's idea that measurements constrain subsequent measurements, which we can prove to be equivalent to a statistical version of Heisenberg's idea that the state collapses after every measurement — [1, Eq. (41)] gives that equivalence with mathematical succinctness: $\mathrm{Tr}[AX\rho_A]=\mathrm{Tr}[AX_A\rho]$, where the notation X_A and ρ_A represents the Lüders operation for the measurement A applied to the measurement X or to the density matrix ρ. The mathematical and philosophical consequences of the collapse of the state can thereby be minimized.

In the six months since the publication of [1], there have been some shifts of emphasis in how I understand and present its material, which I hope will be even slightly more clear by the time q-turn comes around.

[1] Peter Morgan, Annals of Physics, Volume 414, March 2020, 168090, "An Algebraic Approach to Koopman Classical Mechanics", https://doi.org/10.1016/j.aop.2020.168090