Structure of quantum and classical implementations of Popescu-Rohrlich box

Anna Jenčová[1] and Martin Plávala[1,2]

Abstract We construct implementations of the PR-box using non-signaling quantum channels. The main idea of the construction is taken from the general probabilistic theories (GPTs) and the Boxworld theory. We derive a characterization of PR-boxes in GPTs, which we use to construct all quantum qubit channels that maximally violate a given CHSH inequality. We show that all such channels are entanglement-breaking.

Keywords: CHSH inequality, Popescu-Rohrlich box, general probabilistic theories (GPTs)

We investigate the structure of the non-signaling channels such that the CHSH inequality is maximally violated for some choice of channel measurements. Such channels will be called the PR-channels. All the PR-channels obtained so far [1–4] are in fact classical-to-classical and it is natural to ask whether there are some truly quantum non-signaling PR-channels. An important question is the possibility of instantaneous implementation of such channels. Since PR-channels are non-signaling, their instantaneous implementation is not forbidden by special theory of relativity because no information is transferred, yet it is believed that such implementations do not exist.

We present a characterization of the structure of all implementations of the PR-box in the framework of GPTs, especially for theories in which classical and quantum channels play the role of states. It is known that there are GPTs with bipartite non-signaling states and measurements maximally violating the CHSH exist [5, 6]. Moreover in any GPT, the pairs of measurements appearing in such implementations must be maximally incompatible [7–9] and we show how the corresponding bipartite states are constructed from such pairs.

We apply the results to quantum bipartite non-signaling channels, where both parts of the input and output are qubit spaces. We give a full description of all possible pairs of maximally incompatible two-outcome channel measurements and of all qubit PR-channels. In particular, we prove that all these channels are necessarily entanglement-breaking. A quantum channel Φ that maps states on Hilbert space $\mathcal{H}_1$ to states on Hilbert space $\mathcal{H}_2$ is entanglement breaking if for any bipartite state ρ on $\mathcal{H}_1 \otimes \mathcal{K}$ the state $(\Phi \otimes \mathbb{1})(\rho)$ is separable. $\mathbb{1}$ denotes the identity channel.

Our main results are stated informally in the following theorems, exact formulations, proofs and explicit examples of PR-channels are in the preprint [10].

Theorem 1 (Informal, see [10, Theorem 1]). *Let K be a state space of some GPT and let $\{\mathsf{A}_1, \mathsf{A}_2\}$, $\{\mathsf{B}_1, \mathsf{B}_2\}$ be two pairs of two-outcome measurements in the given theory. Let ϕ be a bipartite non-signaling state of the theory such that when ϕ is shared between Alice and Bob, Alice measures either A_1 or A_2 and Bob measures either B_1 or B_2, then the resulting conditional probability distribution maximally violates the CHSH inequality. Then both pairs $\{\mathsf{A}_1, \mathsf{A}_2\}$ and $\{\mathsf{B}_1, \mathsf{B}_2\}$ are maximally incompatible and ϕ is of the form*

$$\phi = \phi_S + \phi^\perp \tag{1}$$

where ϕ_S corresponds to the bipartite state of the Boxworld theory [6] that maximally violates the CHSH inequality and $\phi^\perp$ is a vector such that $(\mathsf{A}_i \otimes \mathsf{B}_j)(\phi^\perp) = 0$ for $i, j \in \{1, 2\}$, i.e., $\phi^\perp$ is a part of the state ϕ that does not contribute to the result of the experiment.

Theorem 2 (Informal, see [10, Theorem 2]). *Let Φ be a qubit PR-channel. Then Φ is an entanglement-breaking channel.*

We believe that our results will bring more insight into the structure of PR-channels, in particular to the question of existence of their instantaneous implementation.

[1]Mathematical Institute, Slovak Academy of Sciences, Štefánikova 49, Bratislava, Slovakia
[2]Naturwissenschaftlich-Technische Fakultät, Universität Siegen, 57068 Siegen, Germany

References

[1] D. Beckman, D. Gottesman, M. A. Nielsen, and J. Preskill, Phys. Rev. A **64**, 052309 (2001), arXiv:0102043 [quant-ph] .

[2] M. J. Hoban and A. B. Sainz, New J. Phys. **20**, 053048 (2018), arXiv:1708.00750 .

[3] M. Plávala and M. Ziman, Phys. Lett. A **384**, 126323 (2020), arXiv:1708.07425 .

[4] C. Crépeau, J. Li, and N. Yang, private communication (2017).

[5] S. Popescu and D. Rohrlich, Found. Phys. **24**, 379 (1994).

[6] J. Barrett, Phys. Rev. A **75**, 032304 (2007), arXiv:0508211 .

[7] N. Stevens and P. Busch, Phys. Rev. A **89**, 022123 (2014), arXiv:1311.1462 .

[8] P. Busch, T. Heinosaari, J. Schultz, and N. Stevens, Europhysics Lett. **103**, 10002 (2013), arXiv:1210.4142 .

[9] A. Jenčová and M. Plávala, Phys. Rev. A **96**, 022113 (2017), arXiv:1703.09447 .

[10] A. Jenčová and M. Plávala, "Structure of quantum and classical implementations of Popescu-Rohrlich box," (2019), arXiv:1907.08933 .