Inverting Non-invertible Maps Through Recovering Maps

Lea Lautenbacher[1], Fernando de Melo[2], and Nadja K. Bernardes[1]

[1] *Departamento de Física, Universidade Federal de Pernambuco, Recife – PE, Brasil*
[2] *Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro - RJ, Brasil*

Abstract.
We investigate the concept of irreversibility in open quantum systems analyzing qubit quantum channels. Also, we propose an approach to classify the non invertibility degree of a quantum map based on the theory of recovering maps. The inverse will be constructed exploring the Petz recovering map. This strategy will be explored to define a domain of Markovianity for these evolutions.

Keywords: invertibility, quantum channels, recovering maps, NM dynamics.

The development of quantum technologies for information processing, communication and high resolution metrology among other applications has renewed the interest in a better understanding of the dynamics of open quantum systems. Analyzing qubits dynamics is the simplest way to understand the fundamental properties of these systems. In this work we analyze the behavior of typical quantum channels : totally dephasing, depolarizing and amplitude damping in the domain where the inverse map is not well defined, and also is not a physical process [1].

According to thermodynamics, the entropy defines the irreversibility degree of a system. If the entropy grows, the system changes its state irreversibly. We want to explore the concept of irreversibility in open quantum systems. The former concept is fully characterized in terms of the monotonicity theorem of the relative entropy [2,3], according to which, after a dynamical evolution, the value of relative entropy between two quantum states can be equal to the value before the evolution if and only if a recovering map exists capable to reverse the physical evolution of both states [4].

Using information recovery maps, here to be the Petz recovery map [4,5,6], we show how the inverse can be easily computed. Also we quantify how non-invertible a dynamics is compared to another. This demonstrates that there exists a difference in the efficiency of recovering the states between the quantum channels and consequently a hierarchy between the dynamics. We also explore the formalism to identify a domain of Markovianity for non-Markovian evolutions [7,8]. Briefly, we simulate a non-Markovian dynamics by a Markovian one [9,10].

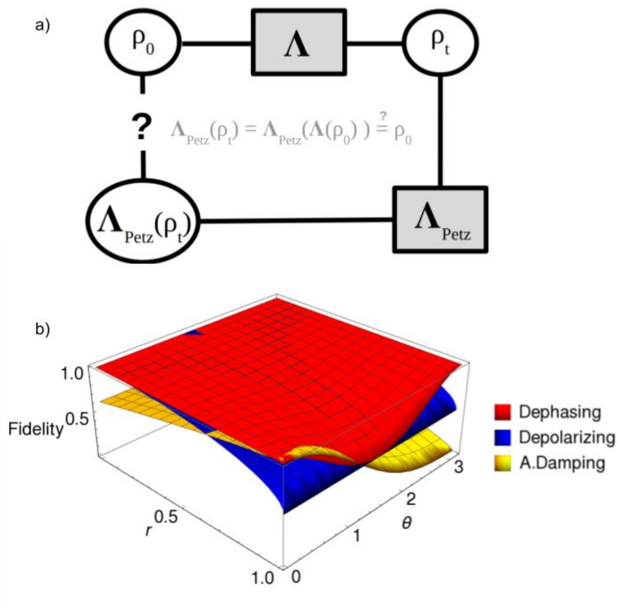

Figure 1 (a) Schematic representation of a system under the action of the Petz recovering map. (b)Fidelity of the state recovered by the Petz in terms of the radial and polar coordinates of the initial random mixed state in the Bloch sphere, azimuthal coordinate due to symmetry fixed in zero.

References

[1] M. A. Nielsen and I. L. Chuang Quantum Computation
and Quantum Information (Cambridge University Press,
Cambridge, 2000).
[2] G. Lindblad. Entropy, information and quantum measurements. Communications
in Mathematical Physics, 33:305–322, 1973.
[3] G. Lindblad. Completely positive maps and entropy inequalities. Communications
in Mathematical Physics, 40(2):147–151, 1975.
[4]Marius Junge et al., Annales Henri Poincare, vol. 19, no. 10, pages 2955--2978, October 2018.
[5]D. Petz. Monotonicity of quantum relative entropy revisited. Reviews in Mathe-
matical Physics, 15(01):79–91, 2003.
[6]D. Petz. Sufficient subalgebras and the relative entropy of states of a von Neumann
algebra. Communications in Mathematical Physics, 105(1):123–131, 1986.
[7]Á. Rivas, S. F. Huelga, and M. B. Plenio, Rep. Prog. Phys.
77, 094001 (2014)
[8] Dariusz Chruściński and Filip A. Wudarski,Phys. Lett. A 377, 1425 (2013).
[9]Dariusz Chruściński et al., Phys. Rev. Lett. 121, 080407 (2018)
[10] Giuliano Benenti et al., J. Phys. B: At. Mol. Opt. Phys. 43 (2010) 215508