Local-Dimension-Invariant Qudit Stabilizer Codes[a]

Lane G. Gunderman[†]

The Institute for Quantum Computing and

University of Waterloo Department of Physics and Astronomy

(Dated: August 4, 2020)

If we're ever going to have a useful and controllable quantum computer we need methods to protect our information from noise. One avenue being attempted is reducing the number of particles that need protection, while still maintaining a large computational space, by using qudits instead of qubits. Experimental realizations of qudit quantum computers have been progressing as well as the theory of making such systems [1][2][3]. These qudits (generally quantum particles with a prime number of levels) will need codes to protect their information as well, however, the primary focus has been on qubit codes so most qudit codes have very challenging relations between the parameters of the code [4]: number of qudits n, number of logical qudits $n-k$, distance of the code d, and number of bases (the local dimension) q. These relations mean that the experimenter might have to not utilize some of their qudits since there are no known codes using their exact relation between those parameters.

Generally codes for qudit systems are found through group theoretic extensions of qubit codes [5] or through using the CSS theorem [6][7] (such as [4][8][9]), however, a few examples of codes are known which have no dependency on their local dimension [10], while still maintaining their distance. My recent work [11] builds off these fortuitous examples, showing that any stabilizer code over q levels can be used as a stabilizer code over any other prime p leveled system. This greatly reduces the restrictions on the relations between parameters. This only says that it's a valid code, but nothing about the distance of these codes. Through a careful analysis of error kinds in the case of non-degenerate codes we have shown the caveat as these *invariant* forms of these codes, stabilizer codes where the commutators for the generators are exactly the zero vector, are only guaranteed to at least preserve the distance of the code beyond some cutoff value p^* which is a function of: n, k, d, and q.

In addition, I identify under what cases the distance of the code can actually *increase* upon being used to protect some system over p levels where $p > q$. We identify another lower cutoff value: p^{**}. These results show that generally if we begin with a code over q levels and use it for a system over p levels the distance: must decrease if $p < p^{**}$, may remain, decrease, or increase for $p^{**} \leq p < p^*$, and must at least remain for $p > p^*$. Lastly, we may construct the logical operators for such codes, thus making these viable choices for protecting qudit information.

In some regards one may consider this a tool somewhat similar in nature to CSS code construction: CSS allows classical to quantum code construction whereas this allows for quantum to quantum code construction. In addition, this work may provide an avenue for determining whether a code is utilizing the qudit space particularly well, beyond the metric of the generalized quantum Hamming bound.

---

[a] Link to this work: `https://journals.aps.org/pra/abstract/10.1103/PhysRevA.101.052343`

[†] lgunderman@uwaterloo.ca

[1] Pei Jiang Low, Brendan M White, Andrew Cox, Matthew L Day, and Crystal Senko. Practical trapped-ion protocols for universal qudit-based quantum computing. *arXiv preprint arXiv:1907.08569*, 2019.

[2] Poolad Imany, Jose A Jaramillo-Villegas, Mohammed S Alshaykh, Joseph M Lukens, Ogaga D Odele, Alexandria J Moore, Daniel E Leaird, Minghao Qi, and Andrew M Weiner. High-dimensional optical quantum logic in large operational spaces. *npj Quantum Information*, 5(1):1–10, 2019.

[3] Rahul Sawant, Jacob A Blackmore, Philip D Gregory, Jordi Mur-Petit, Dieter Jaksch, Jesus Aldegunde, Jeremy M Hutson, MR Tarbutt, and Simon L Cornish. Ultracold polar molecules as qudits. *New Journal of Physics*, 22(1):013027, 2020.

[4] Avanti Ketkar, Andreas Klappenecker, Santosh Kumar, and Pradeep Kiran Sarvepalli. Nonbinary stabilizer codes over finite fields. *IEEE Transactions on Information Theory*, 52(11):4892–4914, 2006.

[5] Daniel Gottesman. Fault-tolerant quantum computation with higher-dimensional systems. In *NASA International Conference on Quantum Computing and Quantum Communications*, pages 302–313. Springer, 1998.

[6] A Robert Calderbank and Peter W Shor. Good quantum error-correcting codes exist. *Physical Review A*, 54(2):1098, 1996.

[7] Andrew M Steane. Error correcting codes in quantum theory. *Physical Review Letters*, 77(5):793, 1996.

[8] Yang Liu, Ruihu Li, Guanmin Guo, and Junli Wang. Some nonprimitive bch codes and related quantum codes. *IEEE Transactions on Information Theory*, 65(12):7829–7839, 2019.

[9] Xiaoshan Kai, Shixin Zhu, and Ping Li. Constacyclic codes and some new quantum mds codes. *IEEE Transactions on Information Theory*, 60(4):2080–2086, 2014.

[10] HF Chau. Five quantum register error correction code for higher spin systems. *Physical Review A*, 56(1):R1, 1997.

[11] Lane G Gunderman. Local-dimension-invariant qudit stabilizer codes. *Physical Review A*, 101(5):052343, 2020.