Unifying entanglement, nonlocality, steerability, and more

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Abstract. We show that a wide array of useful resources that have been previously studied in the context of space-like separation can all be subsumed under one unified notion of nonclassicality. We define a single resource theory which subsumes all of these as special cases, and which allows for comparisons of resourcefulness across arbitrary resource types. This submission relates to References [1] and [2].

Keywords: Bell nonlocality, entanglement, steering, teleportation

A key focus in quantum foundations is the study of nonclassicality, with a special focus on experiments involving space-like separated subsystems. In the language of causality, the key feature of such scenarios is that the subsystems being probed share a classical common cause, but do not share cause-effect channels between them. In such scenarios, quantum theory allows for valuable nonclassical resources which can accomplish tasks which would otherwise be impossible.

The most well-studied types of resources in Bell scenarios are entangled quantum states and boxes producing nonlocal correlations, but there are many others. We develop a resource-theoretic[3] framework which unifies a wide variety of these, including quantum states[4] boxes[5] steering assemblages[6,7] teleportages[8,9], distributed measurements[10], measurement-device-independent steering channels[11], channel steering assemblages[12], Bob-with-input steering channels[13], ensemble-preparing channels, and generic no-signaling quantum channels[14]. All of these can be viewed as instances of distributed quantum channels, where the distinction between different types of resources is given by whether the input and output systems associated with those channels are trivial, classical, or quantum systems:

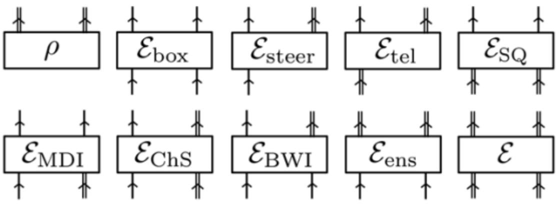

Figure 1 Common types of no-signaling resources, as listed (in order) in the main text. Here, classical systems are represented by single wires and quantum systems are represented by double wires.

We also show that there are some nontrivial types of resources even in the bipartite case which have been overlooked.

We then define a unified notion of nonclassicality that subsumes the standard notions of nonclassicality for each of these resource types as a special case. Our key idea is to define a single *resource theory*[3] wherein the notion of resourcefulness is defined by allowing all and only local operations and shared randomness for free. We show that this simple idea allows one to quantify the resourcefulness of any no-signaling resource of any type, and even to compare nonclassicality *across distinct types.*

We demonstrate the utility of this framework by proving a variety of theorems which unify conceptual results and technical tools which were previously known to hold only for one (or for a small subset) of these resource types. For instance, we prove that any resource of any type can have its nonclassicality quantified in a measurement device-independent[15] manner—that is, without requiring any well-characterized quantum measurement devices. Additionally, we define a hierarchy of tests (each of which is a semidefinite program[7]) which allows one to witness the nonclassicality of any valuable resource of any type, and we define monotones which can be evaluated on resources of arbitrary types.

To concretely illustrate the power of our methods, we completely characterize the relative nonclassicality of five previously studied resources of four different types. We study these both by characterizing all possible conversions between them, and by using a type-independent monotone that we define.

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