## Work Estimation and Work Fluctuations in the Presence of Non-Ideal Measurements

New Journal of Physics, Volume 21, November 2019 - https://doi.org/10.1088/1367-2630/ab4d9d

Tiago Debarba<sup>1</sup>, Gonzalo Manzano<sup>2</sup>, Yelena Guryanova<sup>3</sup>, Marcus Huber<sup>3</sup>, and Nicolai Friis<sup>3</sup>

<sup>1</sup>Departamento Acadêmico de Ciências da Natureza, Universidade Tecnológica Federal do Paraná (UTFPR), Avenida Alberto Carazzai 1640, Cornélio Procópio, Paraná 86300-000, Brazil.

<sup>2</sup>International Center for Theoretical Physics, Strada Costiera 11, Trieste 34151, Italy

<sup>3</sup>Institute for Quantum Optics and Quantum Information - IQOQI Vienna, Austrian Academy of Sciences, Boltzmanngasse 3, 1090 Vienna, Austria

**Abstract:** From the perspective of quantum thermodynamics, realisable measurements cost work, and result in measurement devices that are not perfectly correlated with the measured systems. We investigate the consequences for the estimation of work in non-equilibrium processes and in the fundamental structure of the fluctuations when one assumes that the measurements are non-ideal. We show that obtaining work estimates and their statistical moments at finite work cost implies an imperfection of the estimates themselves: more accurate estimates incur higher costs.

**Keywords:** quantum measurements, quantum thermodynamics, work estimations

From a thermodynamic point of view, acknowledging the energetic cost of measurements is crucial, e.g., for a complete understanding of Maxwell's demon or Szilard's engine [1, 2]. The work-cost of those measurements that are ideal and projective has been investigated by means of the work-value of measurement outcomes [3, 4, 5]. However, a common observation among Refs. [6] is that the benefits derived from using measurements as sources of free energy are either matched or surpassed by the corresponding costs. The energy delivered by measurements is not free of charge and must be supplied to realise the measurement. It was recently shown in [7] that ideal projective measurements require one to prepare the measurement apparatus in a pure initial state. The third law stipulates that such zero-entropy states can only be prepared consuming an infinity amount of resources (see e.g., [8]). It is precisely these considerations that become conceptually important when the purpose of the measurement is to assess the energy consumption itself.

Significant focus in quantum statistical mechanics has been dedicated to the quantification of work and its fluctuations in thermodynamic processes [9, 10, 11, 12]. Studies have also looked at the two-point measurement (TPM) scheme (one of the most prominent approaches for estimating work in an out-of-equilibrium process) [13] in the context of Jarzynski's and Crooks' fluctuation relations [14]. In this work we revisit these concepts and investigate the consequences for these quantities when one does *not* assume ideal measurements. We explicitly show how the average work of the ideal TPM is modified and discuss the operational meaning of the corresponding estimates. We show that while Jarzynski's equality can be maintained exactly at the expense of a correction that only depends on the system's Hamiltonian, the more general relation due to Crooks (as well as related results linking irreversibility and dissipation [15]) no longer hold in the presence of non-ideal measurements. Our results provide a quantification of the cost of obtaining information about work as well the trustworthiness of it. References

- [1] Harvey Leff and Andrew F. Rex, eds., Maxwell Demon 2: Entropy, Classical and Quantum Information, Computing (Institute of Physics, Bristol, 2003)
- [2] K. Mayurama et al. Rev. Mod. Phys, 81:1–23, 2009
- [3] T. Sagawa and M. Ueda, Phys. Rev. Lett., 102:250602, 2009
- [4] K. Jacobs, Phys. Rev. E, 86:040106(R), 2012
- [5] P. Lipka-Bartosik et al., J. Phys. A: Math. Theor., 51:474001, 2018
- [6] G. Manzano et al., Phys. Rev. Lett., 121:120602, 2018
- [7] Y. Guryanova et al. Quantum, 4:222, (2020).
- [8] L. Masanes et al. Nat. Commun., 8:14538, (2017)
- [9] R. Dorner et al. Phys. Rev. Lett., 110:230601, 2013
- [10] L. Mazzola et al., Phys. Rev. Lett., 110:230602, 2013
- [11] L. Fusco et al., Phys. Rev. X, 4:031029, 2014
- [12] A. J. Roncaglia et al., Phys. Rev. Lett., 113:250601, 2014
- [13] P. Talkner et al. Phys. Rev. E, 75:050102(R), 2007
- [14] M. Campisi et al. Rev. Mod. Phys., 83:771, 2011
- [15] J. M. R. Parrondo et al. New J. Phys., 11:073008, 2009