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Compressed Sensing Tomography for qudits: An alternate approach

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The matrix generalizations of Compressed Sensing (CS) were adapted to Quantum State Tomography (QST) previously by Gross *et al.* [Phys. Rev. Lett. 105, 150401 (2010)], where they consider the tomography of n spin-1/2 systems. For the density matrix of dimension $d = 2^n$ and rank r with $r \ll 2^n$, it was shown that randomly chosen Pauli measurements of the order $O[dr \log(d)^2]$ are enough to fully reconstruct the density matrix by running a specific convex optimization algorithm. The result utilized the low operator-norm of the Pauli operator basis, which makes it "incoherent" to low-rank matrices. For quantum systems of dimension d not a power of two, Pauli measurements are not available, and one may consider using SU(d) measurements. Here, we point out that the SU(d) operators, owing to their high operator norm, do not provide a significant savings in the number of measurement settings required for successful recovery of all rank-r states. We propose an alternative strategy, in which the quantum information is swapped into the subspace of a power-two system using only poly$[\log(d)^2]$ gates at most, with QST being implemented subsequently by performing $O[dr \log(d)^2]$ Pauli measurements. We show that, despite the increased dimensionality, this method is more efficient than the one using SU(d) measurements.

Keywords: Quantum State Tomography, Compressed Sensing, Convex Optimization, Low-rank Matrix Recovery

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I. EXTENDED ABSTRACT

In the original work of Compressed Sensing Quatum State Tomography, Gross *et al.* [2] consider the tomography of n spin-1/2 systems. For the density matrix of dimension $d = 2^n$ and rank r with $r \ll 2^n$, it was shown that randomly chosen Pauli measurements of the order $O[dr\log(d)^2]$ are enough to fully reconstruct the density matrix by running a specific convex optimization algorithm. However, these results utilized the low operator-norm of the Pauli operator basis, which are available only in power-of-two dimensional Hilbert spaces. In the present work [1], we propose an alternate CS-QST protocol for states in Hilbert spaces of non-power-of-two dimensions ($d \neq 2^n$), which still achieves the bounds on number of measurement settings $O[dr\log(d)^2]$ presented in [2]. In this alternate protocol, we define a unitary operator W,

$$W = \sum_{i,j}^{d} |i_S\rangle\langle j_S| \otimes |j_A\rangle\langle i_A| + \sum_{i}^{d_1-d} \mathbb{1} \otimes |i_A\rangle\langle i_A| \,, \tag{1}$$

to "move" the quantum information from a d dimensional system to a d_1 dimensional ancilla, where d_1 is a power of two. We prove that, when quantum information is in the ancilla, choosing the optimal value for d_1 and performing the standard CS-QST protocol using simple Pauli measurements on the ancilla will guarentee full recovery from $O[dr\log(d)^2]$ measurements. We show that the unitary operator W, due to its sparsity [3, 4], can be efficiently implemented using only poly$[\log(d)^2]$ single qubit gates at most, which is relatively a small overheard compared to the cost of CS-QST protocol. For states in Hilbert spaces of non-power-of-two dimensions, one may consider performing the standard CS-QST protocol using the SU(d) operators [1]. We point out that the SU(d) operators, owing to their high operator norm, do not provide a significant savings in the number of measurement settings required for successful recovery of all rank-r states. We use numerical simulations to show that the proposed alternate approach outperforms the one using SU(d) operators. In Fig. 1, we compare the Fidelity, which is defined as $F(\rho, \sigma^*) = \mathrm{Tr}\big(\sqrt{\sqrt{\rho}\sigma^*\sqrt{\rho}}\big)^2$, between the estimated (σ^*) and true states (ρ) against the number of measurement settings for SU(31) basis measurements (blue) and alternate approach (orange).

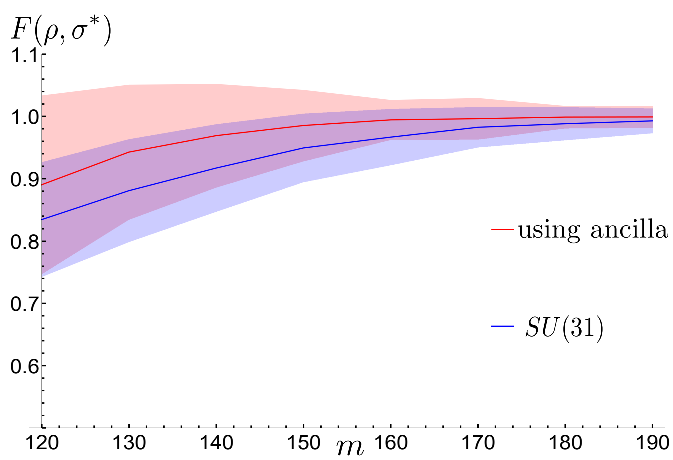

FIG. 1. The fidelity $F(\rho, \sigma^*)$ between the estimated (σ^*) and the true states (ρ) against the number of measurement settings (m) for SU(31) basis measurements (orange) and Pauli measurements on the ancilla (blue) is shown. Fidelity is calculated over 1000 randomly generated 31×31 rank-1 density matrices.

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