MaxEnt estimation applied to quantum states with symmetries

M. Losada[1], F. Holik[2], L. Rebón[3,4] and D. Tielas[3,4]

[1] *Facultad de Matemática, Astronomía, Física y Computación, FAMAF-UNC, Argentina*
[2] *Instituto de Física La Plata, IFLP-CONICET, Argentina*
[3] *Instituto de Física de La Plata, Dpto. de Física, FCE – UNLP, Argentina*
[4] *Departamento de Ciencias Básicas, Facultad de Ingeniería, UNLP, Argentina*

Abstract.
In this work, we present an extension of the MaxEnt principle, adapted to estimate a given unknown quantum state when some a priori information is given in terms of symmetries of the system. Our approach can be applied to very general situations, including symmetries represented by Lie groups and finite groups. We observe that, by using the symmetry information, it is possible to reduce the number of necessary measurements to make a good estimation.

Keywords:
QUANTUM STATE STIMATION, PRINCIPLE OF MAXIMUM ENTROPY, QUANTUM SYMMETRIES

Quantum estimation is a fundamental tool for the experimental characterization of quantum states, a necessary task for several applications in quantum information [1]. A complete estimation can be achieved by performing a quantum state tomography. It consists in obtaining statistical information about an unknown state by measuring in different basis. When a complete set of measurements is available, the unknown state can be completely determined. However, the number of measurements required grows exponentially with the number of particles involved, making this method inviable in practice. In these cases, estimation techniques from an informationally incomplete set become more relevant.

The principle of maximum entropy (MaxEnt), introduced by E. T. Jaynes in the context of the informational approach to statistical mechanics [2, 3], it is one of the most used inference techniques when the available information of the system is incomplete. This principle states that the most appropriate probability distribution to assign to a system is one that is compatible with the available information and maximizes entropy. In this way, it guarantees that the estimated distribution is the least biased. In the field of quantum information, it has also proven to be a viable technique for estimating states when the set of available measurements is informationally incomplete, that is, when it is not possible to perform a tomographic reconstruction of the state [4 - 6]. If additionally, there is some *a priori* information, such as the existence of symmetries or other types of restrictions, the performance of the method can be improved. However, this possibility has not been widely explored in the literature.

In this work, we present a version of the MaxEnt principle, adapted to estimate quantum states using a priori information given in terms of symmetries of the system [7]. Our approach can be applied to very general situations, including symmetries represented by Lie groups and finite groups. We observe that using the symmetry information it is possible to reduce the number of necessary measurements to obtain a good estimation.

The general idea is to rewrite the symmetry constraints in terms of mean values of auxiliary observables, built from the generators of the Lie algebra of the symmetry. I n this way, the standard MaxEnt technique can be adapted to incorporate these extra mean value constraints, to obtain a state estimation which satisfies the symmetry restrictions and maximizes the entropy. The relevance of this method is that there is a substantial reduction in the number of measurements necessary to obtain an acceptable estimation of the state.

Link to published paper
M. Losada, F. Holik, C. Masri and A. Plastino, "Solutions for the MaxEnt problem with symmetry constraints", Quantum Inf. Process., **18**, 293 (2019); doi.org/10.1007/s11128-019-2399-y
https://link.springer.com/article/10.1007/s11128-019-2399-y

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